1 Reddening and absorption

Suppose you observe a star in both B and V filters. You measure apparent magnitudes of $m_V = 8.95$ and $m_B = 9.63$. You know from spectral measurements that this is a B0 giant star (luminosity class Ia), which you can use to determine its absolute V-magnitude to be $M_V = -6.2$ and its intrinsic color to be $(B - V) = -0.24$. You know from other considerations that the star is located at a distance of 2 kpc from the sun and that it has a mass of 20 solar masses (recall that $1\text{pc} = 3.08 \times 10^{18}\text{ cm}$ and that the absolute magnitude of an object is defined as the unabsorbed apparent magnitude it would have at a distance of 10 pc).

A) Calculate the star’s color excess $E(B - V)$

B) Calculate $A_V$, $\tau_V$, $A_B$, and $\tau_B$

C) Calculate the reddening coefficient $R_V$ and compare it to the Milkyway value of $R_V = 3.1$. Is this dust more or less ”reddening” than standard Milkyway dust?

D) Estimate the hydrogen column density towards this object (assume Milkyway composition)

E) Assume that the absorbing gas/dust is located in a uniform sphere of radius $R$ around the star. Calculate the mass in hydrogen inside the sphere as a function of $R$. At what radius does the mass in absorbing gas exceed the mass of the star?

2 Making a tree

In this exercise you will make a tree. The tree is composed of leaves of uniform size and at random orientation, randomly distributed with some volume number density $n$ inside the tree. Assume the tree is perfectly spherical with infinitely thin branches and a radius $R$. The tree is subject to solar radiation from directly overhead (actually, the angle of incidence in this problem is irrelevant, but vertical radiation makes for easy notation). Assume that the rays are perfectly parallel, making the intensity a delta function in angle, $I = F \delta(\theta - \pi)/(2\pi \sin \theta)$.

A) If you were a tree, what would you want your optical depth to be, in an order of magnitude sense? Look at a tree to see if your guess was correct.

B) To get more quantitative, first determine the average cross section per leaf. Each leaf is assumed to be circular with radius $r \ll R$ and is randomly oriented and perfectly flat. What is the mean effective cross section $\sigma$ of each leaf to the incoming parallel radiation?

C) Derive an expression for the optical depth $\tau(x)$ through the tree as a function of impact parameter $x$ (distance of the light ray from the axis of the tree trunk), as shown in the figure. Express your answer in terms of the optical depth through the center of the tree, $\tau_0 = 2Rn\sigma$. 

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D) With your expression for $\tau(x)$ determine the total power of solar radiation absorbed by the tree. Keep in mind that this is a case of pure absorption. Express your answer in terms of the maximum attainable power that the tree could absorb if it were completely opaque, $W_{\text{max}} = \pi R^2 F$, and in terms of $\tau_0$. This is the the incoming power $W_\perp$. Hint: The integral involved can be solved by simple substitution.

E) The tree has to expend a certain amount of power $W_\perp$ to maintain its foliage. The absolute maximum power the tree can afford to expend on a leaf is $\sigma F$. Why?

F) With this, we can express the power required to maintain a leaf as $\epsilon \sigma F$, with some efficiency parameter $\epsilon \leq 1$. Show that you can express $W_\perp$ in terms of $W_{\text{max}}$, $\tau_0$, and $\epsilon$ as

$$W_\perp = \frac{2}{3} W_{\text{max}} \tau_0 \epsilon$$  \hspace{1cm} (2.1)

Plot the total energy balance of the tree $W_\perp - W_\perp$ for the values $\epsilon = [0.1, 0.5, 0.9]$ as a function of $\tau_0$ in log-log plots. From the plots, determine the value of $\tau_{\text{max}}$ for which the energy gain of the tree is maximized for all three cases (if you would rather use a different method to estimate the optimum optical depth, feel free to do so, but explain your algorithm). You will have to choose plot-ranges that allow you to determine the maximum of each curve. Interpret your results and compare your answers to part A). Which of these three values of $\epsilon$ would most appropriately describe the actual tree you observed for part A)?

Figure 1: Sketch of a spherical tree, subject to plane-parallel irradiation from the sun. Light passes through the tree at different impact parameters $x$. 

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