1 Ryden, Problem 6.3

One of the more recent speculations in cosmology is that the universe may contain a quantum field, called “quintessence”, which has a positive energy density and a negative value of the equation–of–state parameter $w$. Assume, for the purpose of this problem, that the universe is spatially flat, and contains nothing but matter ($w = 0$), and quintessence with $w = -1/2$. The current density parameter of matter is $\Omega_{m,0} < 1$, and the current density parameter of quintessence is $\Omega_{Q,0} = 1 - \Omega_{m,0}$.

A) At what scale factor $a_{mQ}$ will the energy density of quintessence and matter be equal?

B) Solve the Friedman equation to find $a(t)$ for this universe.

C) What is $a(t)$ in the limit $a \ll a_{mQ}$?

D) What is $a(t)$ in the limit $a \gg a_{mQ}$?

E) What is the current age of this universe, expressed in terms of $H_0$ and $\Omega_{m,0}$?

2 Numerical solutions to the Friedman equation

In this problem, you will use a numerical integrator to solve the Friedman equation. A very nice software package to use here is Mathematica, but if you do not have access to it, Maple, MatLab, or IDL will work just as well. If you cannot find any access, please make sure to get in touch with me well before the homework is due to make sure you will get the necessary tools.

A) Starting from the Friedman equation formulated in terms of $H_0$ and the density parameters $\Omega_{w}$, write down the differential equation linking the scale factor $a$ and $t$.

B) Let’s start with an easy solution: Numerically integrate the Friedman equation for the case of a matter dominated universe, $\Omega_{m,0} = 1.0$, $\Omega_{r,0} = 0$, $\Omega_{\Lambda,0} = 0$, $\kappa = 1$, $H_0 = [1 \text{ time unit}]^{-1}$ (i.e., all times are expressed in units of $1/H_0$, which makes everything particularly easy). Log-log-plot your solution in the range $10^{-5} < a < 10$. (These are the limits within which you will need to integrate). Repeat the exercise for $\Omega_{m,0} = 0.3$ and all other values being equal.

C) Now integrate the concordance universe ($\Omega_{m,0} = 0.3$, $\Omega_{r,0} = 8.4 \times 10^{-5}$, $\Omega_{\Lambda,0} = 1 - \Omega_{m,0} - \Omega_{r,0}$) within the range $10^{-5} < a < 10$. Again, log-log-plot your result.