28 — Nuclear Reactions

• Proton-Proton Chain
  - Hydrogen is fused into helium in Sun via **PP Chain**
  - Net effect is to convert four hydrogen nuclei into one helium nucleus, neutrinos and radiation
  - Three variants of PP chain. All begin by forming $^3_4\text{He}$:
    
    $^1_1\text{H} + ^1_1\text{H} \rightarrow ^2_2\text{H} + e^+ + \nu_e$
    
    $^2_1\text{H} + ^1_1\text{H} \rightarrow ^3_2\text{H} + \gamma$
    
    First reaction involves decay of proton into neutron — mediated by **weak nuclear force**. Slowest step in whole chain (takes on average 1 billion years for an individual $^1_1\text{H}$!)
  - First reaction indirectly releases additional energy from positron annihilation:
    
    $e^+ + e^- \rightarrow 2\gamma$
  - PP chain is completed via 3 different variants
  - **PP I** — two $^3_2\text{He}$ combine:
    $^3_2\text{He} + ^3_2\text{He} \rightarrow ^5_2\text{He} + 2^1_1\text{H}$
  - Happens in Sun 69% of time
  - **PP II** — $^3_2\text{He}$ combines with $^4_2\text{He}$:
    
    $^3_2\text{He} + ^4_2\text{He} \rightarrow ^7_4\text{Be} + \gamma$
    
    $^7_4\text{Be} + e^- \rightarrow ^7_3\text{Li} + \nu_e$
    
    $^7_3\text{Li} + ^1_1\text{H} \rightarrow ^2_4^8\text{He}$
  - Happens in Sun 31% of time
  - **PP III** — same as PP II, but beryllium combines with proton rather than electron
    
    $^7_4\text{Be} + ^1_1\text{H} \rightarrow ^8_4\text{Be} + e^+ + \nu_e$
    
    $^8_4\text{Be} \rightarrow 2^2_4^3\text{He}$
  - Happens in Sun 0.09% of time; unimportant for overall energy production, but produces high-energy neutrinos
  - Total energy release by any PP chain is 26.7 MeV; but different amounts come out as $\gamma$ rays and as neutrinos (for I, II & III, neutrinos carry 2%, 4% and 28% of energy)
  - Around 15 million Kelvin, overall energy generation rate by PP chain approximated as
    
    $\epsilon_{pp} \approx \epsilon'_{0,pp} \rho X^2 T_6^4$
    
    where $T_6 = T/10^6 \text{K}$. Temperature dependence is modest

• CNO Cycle
  - Higher-mass stars have higher-temperature cores; dominant energy production is hydrogen burning via **CNO Cycle**
- Net effect is same as PP chain: convert four hydrogen nuclei into one helium nucleus, neutrinos and radiation
- However, reaction also involves carbon, nitrogen and oxygen as catalysts (assist reaction, but are not used up)
- Main cycle:
  \[
  \begin{align*}
  ^{12}\text{C} + ^{1}\text{H} & \rightarrow ^{13}\text{N} + \gamma \\
  ^{13}\text{N} & \rightarrow ^{13}\text{C} + e^+ + \nu_e \\
  ^{13}\text{C} + ^{1}\text{H} & \rightarrow ^{14}\text{N} + \gamma \\
  ^{14}\text{N} + ^{1}\text{H} & \rightarrow ^{15}\text{O} + \gamma \\
  ^{15}\text{O} & \rightarrow ^{15}\text{N} + e^+ + \nu_e \\
  ^{15}\text{N} + ^{1}\text{H} & \rightarrow ^{12}\text{C} + ^{2}\text{He}
  \end{align*}
  \]
- There is also a secondary cycle where last reaction produces $^{16}\text{O}$ and photon, initiating a subcycle that leads back to $^{14}\text{N}$
- The step burning $^{12}\text{N}$ is slowest, and so during operation C is depleted and N enriched. If cycle is interrupted (e.g., by mixing to surface), the depletion/enrichment is ‘frozen in’
- Total energy release is same as PP chain, but neutrino emission different
- Around 15 million Kelvin, overall energy generation rate by CNO cycle approximated as
  
  \[\epsilon_{\text{CNO}} \approx \epsilon_{0,\text{CNO}} \rho X X_{\text{CNO}} T_8^{19.9}\]
  
  Note much higher temperature sensitivity!

- **Triple Alpha Process**
  - After hydrogen burning, helium burns to carbon via *triple alpha process*:
    \[
    ^4\text{He} + ^4\text{He} \leftrightarrow ^8\text{Be}
    \]
    (essentially, 3-body interaction, since Be nucleus is unstable)
  - Total energy release is 7.28 MeV — much smaller than hydrogen burning
  - Around 100 million Kelvin, overall energy generation rate by triple alpha approximated as
    \[\epsilon_{3\alpha} \approx \epsilon_{0,3\alpha} \rho^2 Y^3 T_8^{41.0}\]
    where \(T_8 = T / 10^8\) K. Note huge exponent!

- **Other Reactions**
  - In later evolutionary stages, elements can (possibly) burn all the way through to $^{56}\text{Fe}$
  - Carbon burning via alpha capture:
    \[
    ^{12}\text{C} + ^{4}\text{He} \rightarrow ^{16}\text{O} + \gamma
    \]
  - Oxygen burning via alpha capture:
    \[
    ^{16}\text{O} + ^{4}\text{He} \rightarrow ^{20}\text{Ne} + \gamma
    \]
  - Other reactions exist, some of which are endothermic