Asteroseismology with MESA

Assignment I

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Propagation Diagrams

In the lecture, I used a propagation diagram to argue that p-modes are confined between the turning point $r = r_{\rm t}$ (where $\omega = S_\ell$) and the stellar surface $r = R$. In reality, the modes are not truncated precisely at $r = r_{\rm t}$ — there is a little bit of 'slop' to either side, which our simple analysis glossed over.

Nevertheless, it should be possible to find a strong correlation between the location of the turning point and the location of the innermost radial antinode in the displacement perturbation $y_1 \equiv \delta r/r$ (that is, the innermost local maximum of y_1). In this exercise you will search for this correlation using the same 'solar' model you encountered in the mini lab.

The first step is to tweak the input parameters to ADIPLS so that it calculates the eigenfunction y_1 in addition to the usual frequencies. So, change into the `solar` subdirectory and make the following edits to `adipls.c.in`:

- Add the line

  ```
  4 amde.solar @
  ```

  immediately below the first line; this specifies the file to which the eigenfunctions should be written.

- Change the `nfmode` parameter from 0 to 2 (edit the line *following* the comment line which lists `nfmode`). This indicates we want displacement eigenfunctions written.

Then, re-run `adipls.c.in`. The eigenfunction file `amde.solar` is written as an unformatted (binary) Fortran file. The ADIPLS distribution comes provides an IDL script to read this file, and the structure of the file is described in detail in §8.4 of the ADIPLS user guide *Notes on adiabatic oscillation programme*[1]. For the present purposes, however, I put together a simple Fortran program, `extract_eigfunc` which extracts individual eigenfunctions from the eigenfunction file and writes them out in ASCII format. To compile `extract_eigfunc`, run `make` in the `fortran` subdirectory (you may need to change the definition of the `FC` variable in the `makefile`, to match your Fortran compiler). Then, run it from the `solar` subdirectory using the following command:

```
../fortran/extract_eigfunc amde.solar eigfunc.dat 2 l n
```

[1] see *mesa*`/adipls/adipack.c/notes/adiab.prg.c.pdf`

This will extract the eigenfunctions of the mode with radial order `n` and harmonic degree `l` to the file `eigfunc.dat`. The latter file is a simple table, with columns $x = r/R$, y_1 (the radial displacement) and y_2 (the horizontal displacement).

Once you're able to calculate and extract eigenfunctions, pick around six $\ell = 1$ p-modes spanning the frequency range $1,500\,\mu\text{Hz} \lesssim \nu \lesssim 4,500\,\mu\text{Hz}$. For each mode, extract the eigenfunction and use a plot of y_1 as a function of x to determine the location x_1 of the innermost antinode in the radial displacement (*hint*: it's often easiest to locate the antinode by plotting $\log |y_1|$, and looking for the innermost local maximum).

Finally, create a propagation diagram plotting $\log S_\ell$ as a function of x for the 'solar' model (the data necessary for this plot can be found in `log3.data`, which is the MESA profile for the model). Over this diagram, plot the points (x_1, ω) for each of your selected modes (remembering to convert from the linear frequencies ν given in `solar.freq` to the angular frequencies ω required for the plot). The curve indicates the location of the turning point at each frequency; how well do the antinode locations correlate with it? Do they lie on top of the curve, or are they displaced to larger or smaller radii?

Echelle Diagrams

Create FGONG-format models of two stars: $M = 1\,M_\odot$ approaching the terminal-age main sequence (core mass fraction $X_\text{c} = 0.1$), and $M = 1.1\,M_\odot$ near the zero-age main sequence ($X_\text{c} = 0.6$). To get started, make two of the `template` directory, and build MESA star in each using the `./mk` script. Then, edit the `inlist` files to specify the initial mass and stopping criterion. You'll also want to set things up so that MESA writes out the stellar structure to an FGONG file when it exits. Ask a TA if you get stuck!

Run MESA for the two models, and use the resulting FGONG files as input to ADIPLS (you can use the `adipls.c.in` input file from the previous exercises as the input file). Extract frequencies and plot them in an echelle diagram, figuring out an appropriate choice for $\Delta\nu$ by trial and error (or you could cheat and just look at the frequency separation of same-ℓ, neighboring-n modes in the limit of large n). Compare the large and small frequency separations for the two models; what does this tell you about the differences in the internal structure of the stars?