We outline a novel approach to solving the linearized equations of stellar pulsation, which avoids pathologies encountered in previous formulations. Our approach is implemented in GYRE, a new pulsation code applicable to differentially rotating stars. To demonstrate the strengths of the code, we apply it to calculate the frequency spectrum and eigenfunctions of g modes in a white dwarf model, and to explore the non-uniform splitting of a mode multiplet in a rotating B-type main sequence model.

**Introduction**

The process of solving the linearized stellar pulsation equations lies at the heart of a number of areas of current astrophysical interest—e.g., stellar stability, angular momentum transport, tidal interactions, and most prominently, asteroseismology. The pulsation equations pose a boundary-value problem (BVP), in which boundary conditions at the stellar core and surface can be satisfied only for certain choices of the pulsation frequency \( \omega \); that is, \( \omega \) is an eigenvalue of the system.

Previous techniques for solving the pulsation BVP have almost universally suffered from what we call premature normalization—i.e., adopting a specific normalization choice (e.g., imposing a certain solution value at a certain location) during the search for eigenfrequencies. An example is the method popularized by Castor [1]; one of the boundary conditions is set aside, allowing solutions to be obtained that satisfy only the remaining boundary conditions and normalization condition for arbitrary \( \omega \). The eigenfrequencies are then found by varying \( \omega \) until the discarded boundary condition is also satisfied; if this boundary condition is represented by the equation \( D(\omega) = 0 \), then the eigenfrequencies are the roots of this equation.

Castor's method has been used by many authors, within the computational frameworks of both relaxation (finite differencing) (e.g., [2]) and shooting (e.g., [3]). However, it has the drawback that the discriminant function \( D(\omega) \) can be pathological, often exhibiting singularities. The latter arise from the imposition of the normalization condition at locations where the solution exhibits a zero. Because these locations cannot be predicted a priori, it is very difficult to find a normalization that does not cause an ill-behaved \( D(\omega) \).

Our approach to solving the pulsation equations does not require any choice of normalization during the search for eigenfrequencies. Instead, we cast the equations and all accompanying boundary conditions (but not a normalization condition) as a linear, homogeneous matrix equation

\[ S \omega = D(\omega) \]

where \( S \) and \( D(\omega) \) are matrices whose entries are independent of \( \omega \), except that the discriminant \( D(\omega) \) is zero, because the boundary conditions are not satisfied only for certain choices of the pulsation frequency \( \omega \). An example is the method popularized by Castor [1] and shooting (e.g., [3]). However, it has the drawback that the discriminant function \( D(\omega) \) can be pathological; hence, it is very difficult to find a normalization that does not cause an ill-behaved \( D(\omega) \).

**The GYRE Code**

We have implemented this approach in a new code, GYRE, which solves the equations governing adiabatic pulsation in a differentially rotating stellar model. The system matrix \( S(\omega) \) is set up using a multiple shooting technique [4]. To shoot across each shell of the model, we (reasonably) assume that the equation coefficients are constant within the shell. The integration can then be undertaken semi-analytically with the aid of matrix exponentials. The advantage of this approach, which is similar to that in [3], is that it permits arbitrary spatial resolution. Therefore, the eigenfunctions have very short wavelengths pose no especial difficulty to GYRE.

Although GYRE remains under active development, it already performs well on a workstation; we note here that GYRE is parallelized using MPI).

**A High-Resolution Eigenfunction**

To demonstrate the arbitrary spatial resolution that GYRE can achieve, Fig. 2 shows the relative radial displacement eigenfunction \( \Delta R/R \) for a high-order \( g \) mode of the same WD model considered above. A key point to note here is that although hundreds of thousands of points are required to resolve the eigenfunction fully, accurate calculation of the corresponding eigenfrequency \( \omega \) requires only the \( \sim 430 \) original shells of the model! This is thanks to the use of matrix exponentials.

**Summary**

The GYRE code uses a new approach to solving the linearized pulsation equations, which avoids the pathologies introduced by premature normalization. It enjoys arbitrarily high spatial resolution, and is applicable to differentially rotating stars. At the moment, the code is limited to adiabatic pulsation, but we are in the process of incorporating non-adiabatic processes.

Upon its full completion, GYRE will be released into the public domain under an open-source license. Until then, anyone with an interest in using the code should contact one of the authors.

**References**


We gratefully acknowledge support from NSF Astronomy & Astrophysics Grant No. 0986618.