Astronomy 730: Fall 1997 Final Exam (2 hours)

Thursday, December 11th 3:45 - 5:45pm

You should spend approximately one hour on Section A and one hour on Section B. No extra time, 2 hours is it. Give as much relevant information as you can. If you can't remember exactly how to calculate something, or do not have time, describe briefly how the calculation would be done. This is a closed-book exam: calculators OK but no O2s. **DON'T PANIC!!!**

Section A: answer *three* questions out of six.

1) Draw a sketch of the Milky Way as seen edge-on. Indicate, with approximate dimensions, the bulge, spheroid, thin disk, thick disk, and the Sun's position. What is the rotation speed in the disk? The total luminosity? The mass? What fraction of the light comes from the bulge? from the disk? from the spheroid?

How thin is the thin disk? How thick is the thick disk? Where are metal-poor globulars found? Where are metal-rich globulars? Where are the open clusters? How do the ages and metal abundances of these components compare with each other?

Compare the Milky Way with the Large Magellanic Cloud: what is different, what is similar? How would the stars in a 10 pc cube in the disk of the LMC differ from what is in a similar cube in the Milky Way's disk near the Sun? How would the gas be different?

How is the Small Magellanic Cloud different from the Large Cloud?

2) The potential energy of a Plummer sphere of mass $\mathcal{M}$ and core radius a_P is

$$\mathcal{PE} = -\frac{3\pi}{32}\frac{G\mathcal{M}^2}{a_P}.$$

A globular cluster has luminosity $L = 4 \times 10^6 L_\odot$. A random sample of its stars shows that the dispersion in radial velocities V_r is $\sigma_r \approx 10\,\mathrm{km\,s^{-1}}$; the surface brightness of the cluster can be fit approximately by a Plummer model with $a_P = 10\,\mathrm{pc}$. Assuming that the cluster is spherical, and contains no unseen dark matter, use the virial theorem to estimate its mass. What do you have to assume about the stellar random motions?

3) Describe the assumptions of the simple model for the buildup of heavy elements in the stars and gas of a galaxy. What is meant by:
– a 'one zone' model?
– instantaneous recycling?
– the 'closed box'?

In what ways do the simple models succeed, and how do they fail, in describing the metal abundance that we see in
– the Milky Way's bulge
– the Galactic globular clusters
– disk stars near the Sun?

– dwarf spheroidal galaxies?
– dwarf irregular galaxies?
How do we adjust the model in each of these cases?

The ratio [O/Fe] of oxygen to iron is higher in metal-poor stars (with low [Fe/H]) than in metal-rich stars: what explanations have been given for this fact?

4) How does the optical appearance of a galaxy change along the sequence from S0 to Sa, Sb, Sc, Sd, Sm, and Irr? How does the gas content differ?

Figure 5.22 below shows a number of galaxy rotation curves. How do the amplitudes and shapes of the rotation curves differ between the Sab and Sb galaxies, and the Scd, Sd and Irr systems? What do they tell us about the mass in these galaxies, and about its distribution? What would the global profile of the Hi gas look like for the Sb galaxy? For the Irr galaxy?

What mass does the rotation curve imply for the Sb galaxy? For the Scd system? For the Irr galaxy? The distances to these galaxies have been calculated from their redshits, assuming that $H_0 = 75\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. If in fact H_0 is only half as large, how do the masses change? How do the luminosities change? How does $\mathcal{M}/L$ change?

What is the Tully-Fisher relation? The surface brightness I(R) of a galaxy disk is often close to the form

$$I(R) = I_0 \exp[-R/h_R];$$

h_R is the exponential scale length. Show that the disk's luminosity is $L = 2\pi I_0 h_R^2$. Ignoring the bulge, explain why we might expect the mass $\mathcal{M}$ of a spiral galaxy to follow approximately

$$\mathcal{M} \propto V_{max}^2 h_R,$$

where V_{max} is the peak rotation speed, and hence that if the ratio $\mathcal{M}/L$ and the central surface brightness I_0 are constant, we have $L \propto V_{max}^4$.

5) Using the diagram, show that the star S recedes from us at a speed

$$V_r = R_0 \sin l \left(\frac{V(R)}{R} - \frac{V_0}{R_0} \right) .$$

What are approximate values for the Sun's Galactocentric distance R_0 and its rotation speed V_0? On a plot of V_R against Galactic longitude l, sketch the regions where you would expect to find gas, assuming that $V(R)/R$ decreases with radius. Indicate which emission comes from the inner Galaxy, with $R < R_0$, and which is from beyond the Solar circle.

Where is the *tangent point* along a line of sight at longitude l? Why does the velocity V_r have a minimum or maximum value there? Show that at the tangent point,

$$R = R_0 \sin l, \quad \text{and} \quad V(R) = V_r + V_0 \sin l.$$

How do we use this method to find the rotation curve of the inner Galaxy? What are some of the possible sources of error?
How is the rotation curve of the outer Galaxy found?
Sketch the rotation curve for the Galaxy out to $2R_0$.

Once we know the rotation curve, we can use it to map out the distribution of gas in the disk – how is this done? What does the distribution of Hi gas look like in the Milky Way? How does it differ from the distribution of H_2 gas?

6) When can we use the *epicycle* approximation for the motion of a disk star? Sketch the motion of the star and its guiding center. What is meant by the epicyclic frequency κ? If the rotation curve is approximately flat, $V(R) = V_0$, use the formula

$$\kappa^2(R) = \frac{1}{R^3}\frac{d}{dR}[(R^2\Omega)^2]$$

to calculate the epicyclic frequency at radius R. Sketch the curves showing how $\Omega(R) = V(R)/R$, $\Omega - \kappa/2$ and $\Omega + \kappa/2$ vary with radius.

What are the inner and outer Lindblad resonances of a spiral pattern? Why are they important? What explanation does the Density Wave Theory give for the fact that two-armed spirals are more common than those with 3 or 7 arms?

In a spiral galaxy, dust lanes are observed on the inner (concave) sides of the spiral arms. What does this tell you about the pattern speed of the spiral? Explain.

Two-armed spiral structure in galaxies is always observed to trail behind the direction of rotation: give 2 physical reasons (*i.e.*, apart from the reluctance to publish results in contradiction of 'established fact') why this might be so.

Section B: answer *one* question out of three.

1) Suppose the force of gravity did not fall off as the square of the distance, but as $1/r^k$ for some constant k ($k = 2$ according to Mr. Newton). Show that the potential $\Phi(r) = (k-1)G\mathcal{M}/r^{k-1}$ satisfies the equation $\mathbf{F} = -\nabla\Phi$. The force on star α from all the other stars in an isolated galaxy is then

$$\frac{d}{dt}(m_\alpha \mathbf{v}_\alpha) = -\sum_{\substack{\beta \\ \alpha\neq\beta}} \frac{G m_\alpha m_\beta}{|\mathbf{x}_\alpha - \mathbf{x}_\beta|^{k+1}}(\mathbf{x}_\alpha - \mathbf{x}_\beta).$$

Show that the virial theorem becomes

$$2(k-1) < \mathcal{KE} > + < \mathcal{PE} > = 0, \quad \text{where } \mathcal{PE} = \frac{1}{2}\sum_\alpha m_\alpha \Phi(\mathbf{x}_\alpha).$$

What is meant by the angle-brackets?

Now return to normal gravity, so $2 < \mathcal{KE} > + < \mathcal{PE} > = 0$. As a galaxy forms in the expanding Universe, its gravity pulls matter togather and starts to reverse the expansion. Suppose that the galaxy had formed all its stars while the matter was still expanding, so the Virial Theorem applies during collapse. At the point where expansion reverses to contraction, the velocities are zero; suppose that its density is uniform, $\rho(\mathbf{x}) = \rho_0$, and its radius is a. In terms of its total mass $\mathcal{M}$, what is the potential energy? [If you don't remember the numerical factor or have time to calculate it, just give the dependence on $\mathcal{M}$, a, ρ, *etc.*, , and write 'stuff' for the rest.] What is the total energy?

Once the collapse is over and the galaxy has come to equilibrium, let the density in the final state be ρ_f: what is the radius now? What is the potential energy? What is the total energy? Use the Virial Theorem to relate the initial and final potential energy, and thus show that the radius shrinks by a factor of two, so that $\rho_f = 8\rho_0$.

If the galaxy has been partly gaseous, energy could have been dissipated during the collapse: would that have made the final density higher or lower? Why?

2) Why do we think that there is 'dark matter' in the Milky Way? Why do we believe that it is in a fairly round halo, rather than in the disk like most of the stars and gas?

When light passes within distance b of a mass $\mathcal{M}$, it is bent by an angle

$$\alpha = \frac{4G\mathcal{M}}{bc^2}$$

Show that light from a star at distance d_s which is directly behind a 'lensing' mass $\mathcal{M}$ at distance d_L from the observer is spread into an 'Einstein ring' of angular radius θ_E where

$$\theta_E{}^2 = (b/d_L)^2 = (4GM/c^2)(d_{LS}/d_L d_S)$$

when the angles α, θ_E are small.

How big is θ_E in arcseconds, if $\mathcal{M} = \mathcal{M}_\odot$, $d_L \approx 30\,\mathrm{kpc}$ and $d_S = 60\,\mathrm{kpc}$ (*e.g.*, for stars in the LMC lensed by objects in the Milky Way's halo)? The image of a source S will be distorted and brightened if it is behind the lens and within angular distance θ_E.

Show that for any given star (fixed d_S), the lensing cross-section $\pi(d_L\theta_E)^2$ is largest when the lens L is halfway between the source and the observer.
If the 'lenses' are stars or black holes in the Galactic halo, they move at speeds of about 250km/sec; if their mass $\mathcal{M} \sim \mathcal{M}_\odot$, how long does it take for a lens at $D_S = 30$kpc to move across the sky by an angular distance equal to θ_E?
What area of the sky lies within an angle θ_E of this lens during the course of a year?
What range of masses $\mathcal{M}$ would cause a lensed star to brighten for at least a week, so that if you observe once per night you have a good chance of seeing it happening?

How might you be able to distinguish the effect of gravitational lensing from intrinsic variability of the star?

3) Show that a star of mass $\mathcal{M}$ which passes at speed V within distance b of another star with mass m acquires a speed

$$V_\perp = \frac{2Gm}{bV}$$

normal to its original direction of motion, as long as

$$b \gg r_s = \frac{2G(\mathcal{M}+m)}{V^2}.$$

Why is this criterion needed?

Show that if the star $\mathcal{M}$ travels through a region where there are n stars of mass m per unit volume, after time t it acquires a perpendicular speed

$$< \Delta V_\perp^2 >= \int_{b_{min}}^{b_{max}} nVt\left(\frac{2Gm}{bV}\right)^2 2\pi b db$$

— why the squared term in the integral? Show that $< \Delta V_\perp^2 > \sim V^2$ after a time

$$t_{relax} = \frac{V^3}{8\pi G^2 m^2 n \ln(b_{max}/b_{min})}.$$

What value is usually taken for $\Lambda = b_{max}/b_{min}$?
For the Galaxy with $R \approx 20\,\mathrm{kpc}$, and random stellar speeds $V \sim 30\,\mathrm{km\,s^{-1}}$, what is the crossing time? If $m \approx 0.5\mathcal{M}_\odot$, How long is the relaxation time?
For the core of a globular cluster, about 1 pc in radius, random speeds are $\sim 10\,\mathrm{km\,s^{-1}}$; if there are $N \sim 10^4$ stars in the core, with $m \approx 0.5\mathcal{M}_\odot$, what is the relaxation time?
What effects does two-body relaxation have on a cluster which includes stars with a range of masses? How can two-body relaxation explain the steep cusps at the centers of some globular clusters (see figure)?

4) A star is moving in the potential $\Phi(\mathbf{x}, t)$ which is steady in a reference frame rotating with constant angular speed Ω; an observer in the inertial frame sees the star move with speed $\mathbf{v} = d\mathbf{x}/dt$. Explain why the velocity $\mathbf{v'}$ seen by an observer in the rotating frame is

$$\mathbf{v}' = \frac{d\mathbf{x}'}{dt'} = \mathbf{v} - \Omega \times \mathbf{x}.$$

Use this to show that

$$E_J \equiv \frac{1}{2}\mathbf{v}'^2 + \Phi(\mathbf{x}') - \frac{1}{2}(\mathbf{\Omega} \times \mathbf{x}')^2 \frac{1}{2}\mathbf{v}'^2 + \Phi_{eff}(\mathbf{x}')$$

is constant along the star's path.

When the rotating system consists of two point masses $\mathcal{M}$ (a galaxy) and m (a globular cluster), separated by distance d, what is the orbital frequency Ω? Sketch the effective potential Φ_{eff} along the line joining the two masses. What is Φ_{eff} at a point which is at distance x from m in the direction towards $\mathcal{M}$?

If $m \ll \mathcal{M}$, show that stars bound to m must remain within distance r_J of it, where

$$r_J \approx D \left[\frac{m}{3\mathcal{M}}\right]^{1/3}.$$

If a cluster follows an eccentric orbit, all stars which wander further than the distance r_J at the pericenter are likely to be stripped from the cluster. Is a cluster more likely to lose low-mass stars in this way, or higher-mass stars? Why?