Astronomy 730: Galaxies Spring 2006

Computer Project Phase 4: due Friday April 14

Again, use units in which the Plummer model parameters are $a_P = 1 = M$, and $G = 1$.

1. Find the angular speed Ω of a circular orbit in the Plummer model at radius r, and the epicycle frequency κ.

2. What is the angular momemtum of a circular orbit at radius 1? Use your program to integrate an orbit with this angular momemtum that starts at $x = 1.3, y = 0$ with $v_x = 0$ and $v_y > 0$, for about 3 cycles – it should look like Figure 3.9.

3. Now think about how to compare this orbit with what you expect from the epicycle approximation. Convert the x, y pairs describing your orbit to polar R, ϕ. (Why didn't you want to do the integration in R, ϕ? It's the same reason that you can't use WIYN to track a star near the zenith.)
Now transfer to a frame rotating at the speed of the circular orbit at $R = 1$. This is the path of the guiding center – following a circular orbit with the same angular momentum as your particle. Plot the path of the particle in this rotating frame. It should be a near-ellipse around the guiding center. What are the minimum and maximum R-values? Which one deviates more from $R = 1$? Why? (Look at Figure 3.7.)

4. Now set up several particles with guiding centers at $R_g = 1$ with the same epicyclic amplitude X, so as to make an oval shape like Fig 5.28. At the initial time $t = 0$, pick e.g. 15 guiding centers evenly spaced in azimuth $0 \leq \phi_{gc} \leq 2\pi$. Set up the particles on their epicycles following the prescription on page 210 of the text (set $X = 0.2$, you get to pick m):

$$R = R_g + X \cos[\kappa t + m\phi_{gc}] \ , \quad \phi = \phi_{gc} - \frac{2\Omega}{R_g \kappa} X \sin[\kappa t + m\phi_{gc}] \ ,$$

where $R = R_g + x$ as in Equation 3.68, ϕ is given by Equation 3.72, and $t = 0$. Convert these R, ϕ pairs to x, y and plot the particle positions at the start. Mark the direction of some feature (e.g. the long axis of the ellipse if you picked $m = 2$).

In the Plummer potential, integrate all these particles for time $t_{orbit} = 2\pi/\Omega(R_g)$ around the guiding center. Plot the positions of all the particles on the same plot, for (0, 0.1, 0.2, 0.3, 0.5 and 1.0) $\times t_{orbit}$. You should see a rotating pattern. If the epicycle approximation was exact, your feature would have moved by an angle $(\Omega - \kappa/m)t$ from its initial position. Mark this angle on each of your plots and comment on how close the epicycle approximation comes to the result of your integration.

5. *Only if you are really curious about how epicycles work*: set up particles on guiding centers at $R_g = 0.8$ and $R_g = 1.2$ in the same way as for part 4. (You need to change $\Omega(R_g)$ and κ to be right for the new guiding center.) Integrate particle paths for all 3 rings of particles to see how the fact that the pattern speed $(\Omega - \kappa/m)$ depends on radius causes a spiral to form.