

## Astronomy 730: Galaxies Spring 2006

### Computer Project Phase 3: due 4pm, Wednesday March 22

Again, use units in which the Plummer model parameters are  $a_P = 1 = M$ , and  $G = 1$ .

1. We can calculate the positions of the shells at any time. Suppose all the particles start with  $(x > 0, v_x = 0)$ , and the period of a particle reaching out to  $x_{max}$  is  $P(x_{max})$ . Then at time  $t > 0$ , particles that have made exactly  $n$  orbits will form a shell at position  $x_n > 0$  given by  $nP(x_n) = t$ . Inside that is a shell with particles that have made one more orbit:  $(n + 1)P(x_{n+1}) = t$ , and so on. On the negative  $x$ -axis are shells of particles that have made a half-integer number of orbits. The shells are interleaved, such that  $\dots x_{n+1} < x_{n+1/2} < x_n < x_{n-1/2} \dots$ , as you saw in the  $x, v$  plots of Phase 2.

a) Use the results from Phase 1, part 3 to answer the following questions for each of the Plummer model, the point-mass potential and the constant-density model:

If particles are dropped from rest at a whole range of  $x$  positions at  $t = 0$ , and shell  $n = 3$  is at now radius 3, what is the time  $t$ ? Where should the other shells be, for values of  $n \leq 20$ ? (This does not have to be very accurate, linear interpolation from what you found before should be fine.)

b) In a dark halo model where the circular velocity is constant with radius, all the angular frequencies vary as  $1/r$  and the periods of orbits through the center are proportional to  $r_{max}$ . When shell  $n = 3$  is at radius 3, where should the other shells be, for  $n \leq 20$ ?

Plot a single graph showing  $x_n$  as a function of  $n$  for all these 4 potentials, plus a graph of  $\log(x_n)$  against  $\log n$  with the same information.

2. Expand your code to handle a 2D calculation, with a potential  $\Phi(x, y)$ . The force subroutine needs  $F_x$  and  $F_y$ , but remember that if  $\Phi = \Phi(r)$  then  $F_{x,y} = (x, y)F_r/r$ , to simplify and save a few nanoseconds.

Test the code by repeating the integration of the two particles in the Plummer potential from Phase 2, part 1, starting them first on the  $x$ -axis with  $y = v_y = 0$ , then on the  $y$ -axis, then on the line  $x = y$ . Explain how you modified your code and what kind of accuracy you think you are getting. How well is the energy conserved?

Take 25 particles at rest with  $x$  coordinates centered at  $x = 2.5$  with a Gaussian distribution with standard deviation  $\sigma = 0.4$ , and Gaussian  $y$  coordinates centered at  $y = 0$  with the same variance. Integrate these from rest, saving the positions and velocities at  $t = 0, 5, 10, 25, 50, 75$  and 100. Calculate the energy and angular momentum  $xv_y - yv_x$  for each particle, at the start and at  $t = 100$ : what is the maximum fractional change? Now integrate another 225 particles, so you have 250 in all: plot the  $x, y$  positions at each of these times, keeping the scales equal on the two axes so as to see the true shape of the 'shells'. Again, be kind to the trees and put all the plots in small windows on a single page.

Repeat the integration and plots when all the particles start with velocities  $v_x = 0$  and  $v_y = 0.15$ . Why do the shells appear to 'rotate' around the center as time passes?

Which machine did you use for your calculations? Roughly how much CPU time did they take?