

**Computer Project Phase 1: due 3pm, Friday February 3rd**

1) The Plummer model (Problem 3.2 and Equation 3.11 in the book) with potential

$$\Phi(r) = -\frac{GM}{\sqrt{r^2 + a_P^2}}$$

is often used to represent ‘softened gravity’ in  $N$ -body programs. Find the radial force  $F_r$  and use Poisson’s equation to find the density. What is the total mass of the model? What is its central density? This model has too little mass at large radii, and too smooth a central profile, to represent real galaxies.

To simplify, measure masses in units of  $\mathcal{M}$  and lengths in multiples of  $a_P$ , and choose the time unit so that  $G = 1$ . What is your time unit in Myr, if  $\mathcal{M} = 10^6 \mathcal{M}_\odot$  and  $a_P = 1$  pc (typical numbers for a globular cluster)? What is it if  $\mathcal{M} = 10^{11} \mathcal{M}_\odot$  and  $a_P = 5$  kpc, which is more typical of a large galaxy?

Plot the Plummer potential and the potential of a point mass  $\mathcal{M}$  for  $0 < r < 5a_P$ . Your  $y$ -axis should cover something like  $-5 < \Phi < 0.05$  – don’t try to plot the point mass potential all the way to  $-\infty$ . You may want to use the plotting package ‘supermongo’, invoked by typing “sm”. It wants input data as a series of columns separated by spaces or tabs.

2) Integrate the equation of motion for a particle that starts at rest at radius  $r_{max}$  and falls through the center: what is the period  $P$  of its orbit? Plot  $P(r_{max})$  for  $0 < r_{max} < 5a_P$ . (The radial coordinate  $r^2 = x^2 + y^2 + z^2$ ; you can *e.g.*, take  $y = z = 0$  and look at orbits along the  $x$ -axis.)

Routines from Numerical Recipes are available on the linux system. The Fortran version is in /usr/local/recipes.f and the C version is in /usr/local/recipes.c-kr. Both of those directories have a simple Makefile that you can use as a template for creating a Makefile for your own programs. You have two options here. You can use routines from Chapter 4 to do the 1-D integral for the period, being careful about the square-root singularity (§4.4). Or you can integrate the equation for  $(x, \dot{x})$  with routines from Chapter 16, and interpolate to find the period. (You will need the second method for the next step of the computer project.) Remember that you only have to integrate 1/4 of the orbit because it is symmetric.

3) Plot on the same graph as a function of  $r_{max}$

- a) the period of a radial orbit in the Plummer model;
- b) the period of a radial orbit in a uniform distribution of matter with density equal to the central density of the Plummer model. (Write down the equation for the mass within radius  $r$  in a sphere of constant density  $\rho_0$  – what is the radial force?)
- c) four times the free-fall time from radius  $r_{max}$  onto a point mass  $\mathcal{M}$  (this you can figure out from Kepler’s laws).

Two particles are released into each of these potentials, from radii of 3 and 4. How many oscillations has the particle from radius 3 made, when the one dropped from radius 4 has made 5 complete orbits?

In the next part of the project you will drop a clump of particles with slightly differing values of  $r_{max}$  into these potentials, and watch them spread out as they oscillate back and forth. In which potential do you expect the spreading to be most rapid?

### Computer Project Phase 2: probably due 3pm, Friday February 24th

1. Use an integration routine to follow the orbits of two particles in the Plummer potential, starting with zero velocity from  $x = 2$  and  $x = 5$ , for 5 time units. What are  $(x, v_x)$  for the two particles? How accurate do you think your answers are? Check your values with other people to make sure you got the integration routine working properly.

Bulirsch-Stoer routines are good for orbit problems (read why in Numerical Recipes), but this is a simple problem and almost any method will work. Numerical integrators deal with a set of first order equations, so you need to write a subroutine which gives  $(\dot{x}, \dot{v})$  in terms of  $(x, v)$ . You can calculate for the two particles simultaneously by giving the arrays dimension 4 instead of 2; this helps when developing the program further to handle more particles.

2. Find a random number generator (*e.g.*, RAN1 from Numerical Recipes) to produce numbers distributed uniformly between 0 and 1. Then use the subroutine GASDEV or another method to generate random numbers with a Gaussian distribution with zero mean and unit variance.

Use this routine to generate 25 numbers with a mean of 2.5 and standard deviation of 0.4 (multiply the output by 0.4, then add the 2.5). Place particles at these  $x$ -coordinate values, with zero velocity, and integrate their motion up to time 100, saving the output  $(x, v_x)$  every 5 units in files suitable for your plotting program.

a) Plot the  $x$ -values for all the particles against time to produce a plot like this:

Your plot should have “shells”, where the particles pile up in  $x$ , where the particles are reaching the outer limits of their orbits and then reversing direction. The shells look better with 100 particles, but debug your program with 25 first!

b) At each of the times 0, 5, 10, 25, 50, 100, make a plot of the  $(x, \dot{x})$  values. Be kind to the trees and do these as small panels on a single page.

The next step will be 2D models with orbits that are not strictly radial. They should start to look like the picture on page 470 of Binney and Tremaine’s ‘Galactic Dynamics’.