Astro 730 Spring 2006

Homework 4: due Friday April 28 – but I'm out of town that week for all but Tuesday 25th

1) (Problem 4.7, 4.8) From Equation 4.16 show that the mass of stars with metal abundance between Z and $Z + \Delta Z$ is

$$\frac{d\mathcal{M}_s(< Z)}{dZ}\Delta Z \propto \exp\{-[Z(t) - Z(0)]/p\}\Delta Z.$$

Show that if stars are made from gas that is initially free of metals, so that $Z(0) = 0$, the closed-box model predicts that when all the gas is gone, the mean metal abundance of stars is exactly p.
If the disk gas had $Z = 0.15Z_\odot$ at $t = 0$ when stars first began to form, while $Z(\text{now}) \approx Z_\odot$, and $\mathcal{M}_g(t = 0)/\mathcal{M}_g(t) = 50/13$, show that $p \approx 0.63Z_\odot$. From Equation 4.16, show that that about 20% of low-mass stars should have $Z < Z_\odot/4$ today.

2) (Problem 5.4) According to a study of high-surface-brightness spirals (ApJ 160, 811; 1970), the disks all reach $I_B(0) \approx 21.7\,\text{mag}\,\text{arcsec}^{-2}$; this is *Freeman's law*. How many $L_\odot$ does the central square parsec radiate? If its absolute magnitude $M_B = -20.5$, how many $L_\odot$ does the galaxy emit in the B band? If we ignore light from the bulge, show that the exponential disk must have $h_R \approx 5.5\,\text{kpc}$, while $R_{25} \approx 3h_R$, and 80% of the light falls within this radius. For a low-surface-brightness galaxy with the same total luminosity, but $I_B(0) = 24.5\,\text{mag}\,\text{arcsec}^{-2}$, show that $< 10\%$ of the light comes from $R < R_{25}$.
Now consider many spiral disks with $L_B = 2.5 \times 10^{10} L_{B,\odot}$; the larger the length h_R, the smaller $I(0)$ must be. For $1\,\text{kpc} < h_R < 30\,\text{kpc}$, plot R_{25} (in kpc) against h_R, and R_{25} against $I(0)$. Show that R_{25} is small when h_R is small, rises to a maximum, and declines to zero at $h_R \approx 24\,\text{kpc}$. Explain why galaxies with $I(0)$ more than ten times lower than Freeman's value might have been missed from his 1970 sample. (Very small galaxies are also difficult to study: those with $R_{25} < 30''$, or 6 kpc at $d \approx 40\,\text{Mpc}$, are likely to be omitted from catalogues.)

4) (Problem 5.10: Tully-Fisher) Ignoring the bulge, use Equation 3.20 to explain why we might expect the mass $\mathcal{M}$ of a spiral galaxy to follow approximately

$$\mathcal{M} \propto V_{\max}^2 h_R\ .$$

Show from Equation 5.1 that $L = 2\pi I(0)h_R^2$, and hence that if the ratio $\mathcal{M}/L$ and the central surface brightness $I(0)$ are constant, then $L \propto V_{\max}^4$. In fact $I(0)$ is lower in low-surface-brightness galaxies: show that if these objects follow the same Tully-Fisher relation, they must have higher mass-to-light ratios, with approximately $\mathcal{M}/L \propto 1/\sqrt{I(0)}$.

5) When a spherical galaxy with stellar density $n(r)$ is viewed from a great distance along the axis z, show that the surface density at distance R from the center is

$$\Sigma(R) = 2\int_0^\infty n(r)dz = 2\int_R^\infty \frac{n(r)rdr}{\sqrt{r^2 - R^2}}\ .$$

If $n(r) = n_0(r_0/r)^\alpha$, show that as long as $\alpha > 1$ we have

$$\Sigma(R) = 2n_0 r_0 (r_0/R)^{\alpha-1} \int_1^\infty \frac{x^{1-\alpha}dx}{\sqrt{x^2-1}} = \Sigma(R = r_0)(r_0/R)^{\alpha-1}.$$

(What happens if $\alpha < 1$?) The surface density $\Sigma(R)$ remains finite as $R \to 0$ if the volume density rises less steeply than $n \propto r^{-1}$.

6) Suppose that a galaxy is made from N identical fragments, each of mass $\mathcal{M}$ and size R. In each one, the average separation between any two stars (or dark matter particles) is $R/2$; so the potential energy $\mathcal{PE} \approx -G\mathcal{M}^2/R$, as in Equation 3.54. Use the virial theorem to show that each fragment has energy $\mathcal{PE}/2$, so that while they are well separated and moving towards each other only slowly, the total energy $\mathcal{E} \approx -G\mathcal{M}^2 N/2R$. Long after the merged galaxy has come to virial equilibrium, its energy is $-G(\mathcal{M}N)^2/2R_g$; show that its new size $R_g = NR$, and its density is only $1/N^2$ as large as in the original fragments. Explain why Figure 6.6 supports the idea that giant elliptical galaxies (filled circles) arise by repeated merger.