

Homework 9: Due **Wednesday, Nov 29, 2006**, in class (or turn them in before then to my mailbox, 5th floor, Sterling Hall). Circle the correct option and give a short answer.

1. Suppose the Sun collapsed to a white dwarf without losing any mass. Its radius would shrink from 700,000 km at present, to 7000 km as a white dwarf. How many times larger would its density become?

Use Kepler's 3rd law to find the period in years of an orbit just above the surface of the white dwarf, so that $a = 7,000$ km. Show that this is around ten seconds.

[If a white dwarf rotated hundreds of times per second, then matter on the surface would fly off. Because pulsars spin so fast, we know they must be far smaller than a white dwarf.]

2. What is meant by the *Schwarzschild radius* of a black hole?

What would be the Schwarzschild radius of a black hole with the same mass as Jupiter?

What would be the Schwarzschild radius of the black hole at the Milky Way's center, with a mass of $4 \times 10^6 M_{\odot}$?

Suppose that the Sun collapsed to a black hole. Use Kepler's law to find the period in years of a circular orbit at the Schwarzschild radius. How many seconds is this?

Show that an object in this orbit would have to move at roughly $2c/3$.

[Kepler's law is not really valid in such a strong gravitational field – but this problem shows that objects near the Schwarzschild radius must move close to light speed just to avoid falling in.]

3. a) The Sun orbits the Milky Way at 220km/sec. What fraction of the speed of light is this?

b) The Sun is 30,000 light years from the Milky Way's center, following a nearly circular orbit. How many light years is the circumference of its orbit?

c) Show that the Sun takes about 260Myr (not 280Myr) to circle the Milky Way once.

d) Converting 30,000 light years to AU, use Kepler's formula $P^2(\text{in yr})/a^3(\text{in AU}) = 1/M(\text{in } M_{\odot})$ to show that the mass of the Milky Way inside the Sun's orbit is roughly $10^{11} M_{\odot}$.

e) The most distant clouds of hydrogen gas in the Milky Way's disk are about 100,000 light years from the center. They also orbit at 220km/sec. Use Kepler's formula to show that the mass of the Milky Way inside the orbit of this gas is about $3 \times 10^{11} M_{\odot}$.

f) Most of the stars in the Milky Way are closer to its center than the Sun is. Explain how your answer to part (e) tells us that much of the Milky Way's mass is 'dark matter' and not stars.

g) What do we think the dark matter could be made of, and why?