1) Derive the relationship between \( f_\nu \) and \( f_\lambda \).

2) Using Snell’s Law, show how \( d\Omega \, dA \) changes from a medium of index \( n_1 \) into a medium of index \( n_2 \). You may assume that the boundary is planar (which will be the case for an infinitesimal ray bundle in any case). Then show that the conserved quantity is \( I_\nu / (n^2) \).

2) In the 80's, RCA produced a CCD with a detection efficiency at 4000Å of 60% and a readout noise of 80 electrons. Texas Instruments produced a CCD with a detection efficiency at 4000Å of 30% and a readout noise of 8 electrons. Suppose you want to observe a white dwarf at 4000Å to look for variability on a 1 second timescale (i.e., you make many 1 second integrations). Both CCD’s are available at Kitt Peak National Observatory; in your proposal which CCD would you request? Presume that with the telescope and instrumentation available to you at Kitt Peak, 5000 white-dwarf-4000Å-photons fall on a pixel each second, and ignore any source of noise other than photon statistics and readout noise. Calculate the DQE of each detector for this observation.

4) Consider a \( d_{tel} = 8m \) aperture telescope used for direct imaging with a detector having an average quantum efficiency \( q = 0.65 \) over a bandpass \( \Delta \lambda = 1000\AA \) from 5000 to 6000 Å. This hypothetical telescope is in a superb site with a sky brightness of \( B = 22.5 \) mag/arcsec² and seeing disk diameter of \( d^* = 0.5 \text{ arcsec} \). A zero magnitude star in this bandpass has a photon flux \( F(0) \) of roughly 103 photons/sec×cm²×Å.

   a) Write down a formula for the signal/noise obtained for a star of magnitude \( V \) with a bandpass \( \Delta \lambda \) and exposure \( \Delta t \), assuming that the mean sky background brightness \( B \) is known perfectly (through measurements over a large area) and only contributes noise through the photons it contributes within seeing disk of the star.

   b) How bright must a star be to obtain a signal/noise = 3 (the "limiting magnitude") for \( \Delta \lambda = 1000\AA \) and \( \Delta t = 3600 \text{ sec (one hour)} \)? How many photons are contributed by the sky background and by the star to the seeing disk over this exposure? (This is called the "background limited" regime)

   c) What must the focal ratio of the telescope be to have the seeing disk diameter be smaller than two 27µ CCD pixels?

   d) Now suppose that one has some sort of adaptive optics that reduces the seeing disk to 0.2 arcsec. Calculate the aperture and focal ratio of the telescope that would have the same limiting magnitude in the same bandpass and exposure, and the same ratio of seeing disk to pixel size. Verify that this is still in the "background limited" regime.