Total $E/t = \text{Luminosity}, L$

\[ dE = L(t)dt \]
\[ dE = L_\nu(t)d\nu\, dt \]
\[ dE = L_\lambda(t)d\lambda\, dt \]

$L_n$ = specific luminosity
Flux

\[ dE = f_\nu dA d\nu \, dt \]

\[ dE = f_\nu (4\pi R^2) d\nu \, dt = L_\nu d\nu \, dt \]

\[ \therefore f_\nu = \frac{L_\nu}{(4\pi R^2)} \]

Flux is energy incident on some area \( dA \) of the Earth’s surface. Flux is not conserved and falls off as \( R^{-2} \).

---

Flux

- Flux is measured in Janskys in the radio
- \( 1 \text{Jy} = 10^{-26} \text{W m}^{-2} \text{Hz}^{-1} \)
- In the visible flux is measured in apparent magnitudes

\[ m_1 - m_2 = -2.5 \log_{10} \left( \frac{f_1}{f_2} \right) \]

\[ m = -2.5 \log_{10} \left( \frac{f_1}{f_0} \right) \]
Standard choices for reference flux

• Vega system: apparent Magnitude of Vega = 0 in all bands.
• Convenient, but non-physical
• A-B magnitude system:
  • $F_0=3.63\times10^{-23}$ W m$^{-2}$ Hz$^{-1}$, flat spectrum
  • Agrees with Vega at 548nm (center of V-band)

Interesting magnitudes (V-band)

• Sun: $m=-26.7$
• Full moon: $m=-12.6$
• Sirius: $m=-1.5$
• Naked eye limit: $m=6$
• Brightest stars in Andromeda: $m=19$
• Present day limit: $m\sim29$
• Night sky: $m=21.5$ (best sites, dark time)
• Night sky: $m=18$ (bright time)
Flux: absolute magnitude

\[ m_\lambda - M_\lambda = 5 \log_{10} d - 5 + A(\lambda) \]

\[ \therefore \frac{f_1}{f_2} = \left( \frac{d_2}{d_1} \right)^2 \]

- Absolute magnitude is the apparent magnitude that would be observed at 10 pc.
- \(A\) is the total extinction due to interstellar dust in magnitudes

For small changes in flux

\[ \Delta m = 2.5 \left( \log_{10} (f + \Delta f) / (\ln 10) \right) = 1.086 \Delta f \]

\[ \therefore \frac{f_1}{f_2} = 1 - \Delta f \quad \text{and} \quad \Delta f << 1 \]
Intensity

- Finite size source (subtends a real angle)
- Specific intensity
- Brightness, surface brightness
- Specific brightness
- Units: (Jy sr$^{-1}$) or (W m$^{-2}$ Hz$^{-1}$ sr$^{-1}$) or (erg cm$^{-2}$ Hz$^{-1}$) or (m arcsec$^{-2}$)
- What happens when the source is not resolved?

Intensity

\[ dE = I_\nu(\Omega, \nu, t, p) d\Omega d\nu dt dA \]

Where I will depend on:
- Omega measured in RA and Dec
- \( \nu = \) frequency
- t= Integration time
- P=polarization
- Location where you are receiving the light.
**Observation**

\[ E = \int I_v(\Omega, \nu, t, p) r(\Omega) F(\nu) d\Omega d\nu dt dA \]

\[ E = A \Lambda t \int I_v(\Omega, \nu, t, p) r(\Omega) F(\nu) d\Omega d\nu \]

- E=energy received during measurement
- R=energy from the sky
- F= filter function

3100 Å is the UV atmospheric cutoff

1.1 µ silicon bandgap
Digital Detectors

- By far the most common detector for wavelengths $300\text{nm} < \lambda < 1000\text{nm}$ is the CCD.

CCDs

1. Quantum efficiency is more than an order of magnitude better than photographic plates.
These are silicon fab-line devices and complicated to produce.

CCDs remain physically small compared to photographic plates, but they took over rapidly anyway.
CCDs: How do they work?

- Silicon semiconductors with "gate" structure to produce little potential corrals.

Photons $\rightarrow$ Electrons bumped into conduction band $\rightarrow$ Read out and amplify current $\rightarrow$ A/D convert to DN

`clock` parallel and serial registers
``CTE'' > 0.99999

CCD operation

- At room temperature, electrons in high-energy tail of the silicon spontaneously pop up into the conduction band: "dark current". Cooling the detectors reduced the dark current although at about -120C the quantum efficiency starts to decrease.
- Therefore, CCDs usually are put into dewars with liquid nitrogen cold baths and heaters and the temperature is actively controlled to ~1C.
- Readout speed is typically adjustable--faster readout gives higher readout noise per pixel.
CCDs cont.

- The potential corrals that define the pixels of the CCD start to flatten as e\(^{-}\) collect. This leads first to saturation, then to e\(^{-}\) spilling out along columns.
- The “inverse gain” is the number of e\(^{-}\) per final “count” post the A/D converter.
- One very important possibility for CCDs is to tune the response to be linear.

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What gain do you want?

Example: a SiTe 24μ-pixel CCD has pixel "wells" that hold ~350,000 e-

- 16-bit unsigned integer A/D saturates at 65525DN
- Would efficiently maximize dynamic range by matching these saturation levels:

\[
\frac{350,000}{65,535} = 5.3 \frac{e^-}{DN}
\]

- Note, this undersamples the readout noise and leads to "digitization" noise.

Signal-to-Noise (S/N)

- Signal=\(R \cdot t\) detected e-/second
- Consider the case where we count all the detected e- in a circular aperture with radius \(r\).
• Noise Sources:

\[
\begin{align*}
\sqrt{R_\star \cdot t} & \quad \Rightarrow \text{shot noise from source} \\
\sqrt{R_{\text{sky}} \cdot t \cdot \pi r^2} & \quad \Rightarrow \text{shot noise from sky in aperture} \\
\sqrt{\text{RN}^2 \cdot \pi r^2} & \quad \Rightarrow \text{readout noise in aperture} \\
\sqrt{\left[\text{RN}^2 + (0.5 \times \text{gain})^2\right] \cdot \pi r^2} & \quad \Rightarrow \text{more general RN} \\
\sqrt{\text{Dark} \cdot t \cdot \pi r^2} & \quad \Rightarrow \text{shot noise in dark current in aperture}
\end{align*}
\]

\(R_\star = e^-/\text{sec} \text{ from the source}\)

\(R_{\text{sky}} = e^-/\text{sec/pixel from the sky}\)

\(\text{RN} = \text{read noise (as if \text{RN}^2 e^- had been detected)}\)

\(\text{Dark} = e^-/\text{second/pixel}\)

---

\[\text{S/N for object measured in aperture with radius } r: n_{\text{pix}} = \#\text{ of pixels in the aperture} = \pi r^2\]

\[
\frac{\text{Signal}}{\text{Noise}} = \frac{R_\star \cdot t}{\sqrt{(R_\star \cdot t)^2 + \left[R_{\text{sky}} \cdot t \cdot n_{\text{pix}} + \left(R_{\text{RN}} + \frac{\text{gain}}{2}\right) \cdot n_{\text{pix}} + \text{Dark} \cdot t \cdot n_{\text{pix}}\right]^2}}
\]

\(\text{Noise from sky } e^- \text{ in aperture}\)

\(\text{Readnoise in aperture}\)

\(\text{Noise from the dark current in aperture}\)

\(\text{All the noise terms added in quadrature}\)

\(\text{Note: always calculate in } e^-\)
S/N Calculations

- So, what do you do with this?
  - Demonstrate feasibility
  - Justify observing time requests
  - Get your observations right
  - Estimate limiting magnitudes for existing or new instruments
  - Discover problems with instruments, telescopes or observations

Side Issue: S/N $\Leftrightarrow$ $\delta$mag

$$m \pm \delta(m) = c_o - 2.5 \log(S \pm N)$$

$$= c_o - 2.5 \log\left[S\left(1 + \frac{N}{S}\right)\right]$$

$$= c_o - 2.5 \log(S) - 2.5 \log(1 + \frac{N}{S})$$

$$\delta(m) = 2.5 \log\left(1 + \frac{1}{N/S}\right)$$

$$= \frac{2.5}{2.3} \left[\frac{N}{S} - \frac{1}{2} \left(\frac{N}{S}\right)^2 + \frac{1}{3} \left(\frac{N}{S}\right)^3 - \ldots\right]$$

$$= 1.087\left(\frac{N}{S}\right)$$

Note: in log +/- not symmetric

This is the basis of people referring to +/- 0.02mag error as “2%”
How do you get values for some of these parameters?

- Dark Current: CCD@-120°C < 2e-/pix/hour
  Insb: ~2e-/pix/sec
- RN: CCD: 2 - 6 e-/pix
  Insb: 10 - 25 e-/pix
- R*: for a given source brightness, this can be calculated for any telescope and total system efficiency.
- In practice: Go to the facility WWW site for everything!
Source Count Rates

Example: LRIS on Keck 1
for a B=V=R=I=20mag object @ airmass=1

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>B</td>
<td>1470 e-/sec</td>
</tr>
<tr>
<td>V</td>
<td>1521 e-/sec</td>
</tr>
<tr>
<td>R</td>
<td>1890 e-/sec</td>
</tr>
<tr>
<td>I</td>
<td>1367 e-/sec</td>
</tr>
</tbody>
</table>

To calculate \( R^* \) for a source of arbitrary brightness only requires this table and a bit of magnitude math.

\[
m_1 = c_o - 2.5 \log(I_1) \quad \text{(1)}
\]
\[
m_2 = c_o - 2.5 \log(I_2) \quad \text{(2)}
\]
\[
m_1 - m_2 = -2.5 \left[ \log(I_1) - \log(I_2) \right] \quad \text{(1) - (2)}
\]
\[
m_1 - m_2 = -2.5 \log \left( \frac{I_1}{I_2} \right)
\]
\[
\frac{I_1}{I_2} = 10^{\left( \frac{m_1 - m_2}{2.5} \right)} \quad \text{Let } I_2 \text{ be the intensity for the fiducial } m=20 \text{ object}
\]

\[
I_1 = R^*(m_1) = I_{20} \cdot 10^{\left( \frac{m_1 - 20}{2.5} \right)}
\]
Signal from the sky background is present in every pixel of the aperture. Because each instrument generally has a different pixel scale, the sky brightness is usually tabulated for a site in units of mag/arcsecond$^2$.

\[
R_{\text{sky}}
\]

Example, LRIS in the R-band:

\[
R_{\text{sky}} = 1890 \times 10^{0.4(20-24.21)} = 39.1 \text{ e}^-/\text{pix/sec}
\]

\[
\sqrt{R_{\text{sky}}} = 6.35 \text{ e}^-/\text{pix/sec} = \text{RN in just 1 second}
\]
S/N - some limiting cases. Let’s assume CCD with Dark=0, well sampled read noise.

\[
\frac{R_t}{\left[ R_s \cdot t + R_{\text{sky}} \cdot t \cdot n_{\text{pix}} + (RN)^2 \cdot n_{\text{pix}} \right]^{1/2}}
\]

**Bright Sources:** \((R_s t)^{1/2}\) dominates noise term

\[
S/N = \frac{R_t}{\sqrt{R_s t}} = \sqrt{R_s t} \propto t^{3/2}
\]

**Sky Limited \((\sqrt{R_{\text{sky}} t} > 3 \times RN)\):** \(S/N \propto \frac{R_t}{\sqrt{n_{\text{pix}} R_{\text{sky}} t}} \propto \sqrt{t}
\]

Note: seeing comes in with \(n_{\text{pix}}\) term

---

**What is ignored in this S/N eqn?**

- Bias level/structure correction
- Flat-fielding errors
- Charge Transfer Efficiency (CTE) 0.99999/pixel transfer
- Non-linearity when approaching full well
- Scale changes in focal plane
- A zillion other potential problems