A500 / Homework 2, Spring 2017 / Solutions

All problems refer to material from Lectures 5-6 with reference back to Lecture 2.

1a. [2.5 points] Only in sky-limited and read(detector)-noise limited cases is $S/N \propto n_{pix}^{-1/2} \propto \theta_{PSF}^{-1}$ for unresolved sources.

1b. [2.5 points] No, because if $\theta \gg \theta_{PSF}^{-1}$, then $S/N \propto \theta^{-1}$, i.e., the source is resolved.

1c. [5 points] In sky-limited case: $S/N \propto \frac{\sqrt{t\ \epsilon}}{\theta_{PSF}}$, so that $t \propto \theta_{PSF}^2 \epsilon^{-1}$, where t is the exposure time and ϵ is the system efficiency. In the detector-limited case: $S/N \propto \frac{t\ \epsilon}{\theta_{PSF}}$ so that $t \propto \theta_{PSF}\epsilon^{-1}$.

1d. [10 points] If you take the formal definition from the texts then what you are looking for is an effective efficiency (DQE), of the system if it were source-limited. In this case

$$DQE = \epsilon(1-L)^n \left[1 + \frac{S_{sky}}{S_{obj}} n_p + \frac{RN^2 n_p}{\epsilon(1-L)^n S_{obj} t}\right]^{-1},$$

where ϵ is the nominal system efficiency without and AO system, L is the fractional loss per each of n surfaces, $n_p \propto \theta_{PSF}^2$ is the number of pixels in the aperture, S_{sky} and S_{obj} are sky and object fluxes, and RN is the read-noise (rms). However, it is much simpler and more instructive to consider how the system efficiency changes in the three limiting cases for an unresolved sources:

Source-limited: $S/N = \sqrt{S_{obj}\ \epsilon\ t} \ \propto\ \epsilon$ so $DQE_{sys} = \epsilon(1-L)^n$

Sky-limited: $S/N = \frac{S_{obj}}{\theta_{PSF}}\sqrt{\frac{\epsilon\ t}{S_{sky}}} \ \propto\ \sqrt{\epsilon}/\theta_{PSF}$ so $DQE_{sys} = \epsilon(1-L)^n/\theta_{PSF}^2$

Detector-limited: $S/N = \frac{S_{obj}\ \epsilon\ t}{RN\ \theta_{PSF}} \ \propto\ \epsilon/\theta_{PSF}$ so $DQE_{sys} = \epsilon(1-L)^n/\theta_{PSF}$

1e+f. [5+5 points] Refer to results in 1c. If sky-limited, then $\frac{\sqrt{\epsilon_1}}{\theta_1} = \frac{\sqrt{\epsilon_2}}{\theta_2}$, where sub-script 1 is without AO and sub-script 2 is with AO. It follows $\frac{\epsilon_1}{\epsilon_2} = \left(\frac{\theta_1}{\theta_2}\right)^2 = (1-L)^{-4}$, or $L = 1 - \sqrt{\frac{\theta_2}{\theta_1}}$ to break even. For $\theta_1 = 0.8,\ 0.5$ arcsec and $\theta_1 - \theta_2 = 0.1$ arcsec, $L = 0.065,\ 0.106$, respectively. When detector-limited, the same argument yields $L = 1 - \left(\frac{\theta_2}{\theta_1}\right)^{1/4} = 0.033,\ 0.054$ again for $\theta_1 = 0.8,\ 0.5$ arcsec.

1g. [5 points] Refer to results in 1e+f. With 3% coatings you are better off with the WTTM (up to $\theta_{PSF} = 1.7$ arcsec), but in the detector limit you just barely break even at $\theta_{PSF} = 0.8$ arcsec.

1h. [5 points] DQE_{sys} changes by the relevant ratios of system efficiency and PSF. Background-limited: $(1-0.03)^4(0.8/0.7)^2 = 1.156$, or 15.6% gain. Detector-limited: $(1-0.03)^4(0.8/0.7) = 1.012$, or 1.2% gain.

2a. [10 points] Ignore extinction since no airmass is given for the observation. Extra credit if you do. Seeing is irrelevant for this problem. The goal is to determine sky flux in terms of detected electrons pixel^{-1} sec^{-1} $\equiv N_e$, and then determine what exposure time, t, is required such that $\sqrt{N_e\ t}\ >\ 3\ RN$, where RN is the rms read-noise (in electrons).

In the I band, $S_{sky} = 19.9$ mag arcsec^{-2} with no moon, equivalent to 27.96 μJy given the zero-point of 2550 Jy. Calculating the effective telescope collecting area $A = (1-0.17) \times \pi \times (3.5/2)^2 = 7.99$ m^2, the total system throughput (efficiency) $\epsilon = 0.70 \times 0.985^8 \times 0.89^3 = 0.44$, and a band-width of 0.19 nepers yields $N_e = 280.3$ electrons sec^{-1} arcsec^{-2}. For pixel solid-angle of (0.11 arcsec)^2 = 0.012 arcsec^2, $N_e = 3.39$ electrons sec^{-1} pix^{-1}. Given $RN = 9$ electrons, this yields $t > 2151$ sec to be sky-limited, close to 3.5 minutes.

2b. [5 points] Refer to 2a. By narrowing the band-width of the filter to 1%, there is a factor of 19 drop in N_e (the I band is a 19% filter, i.e., $\Delta\lambda/\lambda = 0.19$). This means the exposure time to be sky-limited goes up by a factor of 19, namely $t > 4064$ sec, or 68 minutes – a bit more than an hour. Ouch.

2c. [10 points] Again: ignore extinction; seeing is irrelevant. The same calculation is made, now with $\epsilon = 0.7 \times (0.98)^8 \times 0.89^3 = 0.42$, pixel solid-angle of 0.010 arcsec^2, and band-width of 0.157 nepers. In the K-band, $S_{sky} = 12.9$ mag arcsec^{-2} with no moon, equivalent to 4635 μJy with a zero-point of 670 Jy. This is equivalent to $N_e = 368$ electrons sec^{-1} pix^{-1}, yielding $t > 9.8$ sec to be sky-limited with a detector noise of 20 electrons (rms; Fowler sampled).

2d. [5 points] Refer to 2c. The Brγ filter is again about a 1% band-pass. The band-pass has been narrowed by a factor of $\sim$16 so that $t > 153$ sec to be sky-limited.

3. [30 points] The data you were asked to analyze is from the upgraded Bench Spectrograph CCD (STA1), courtesy of Marsha Wolf. The task is to carry out a standard photon propogation analysis with these data to relate the standard deviation (σ) measured in DN at a give count level, also measured in DN. At low count levels this relationship should be flat, with $\sigma_{\rm DN} = {\rm RN}/\gamma$, where RN is in electrons and γ is the gain in electrons per DN. At high count levels the relationship should be linear in the log, i.e., $\log\sigma = a \log {\rm counts} + b$, because $\sigma_{\rm DN} = \sqrt{F/\gamma}$ where F is the count level (in DN). Because of the sqaure-root, the slope is exactly $a = 0.5$. The zeropoint of the linear relation, b, is related to the gain: $\gamma = {\rm dexp}\ -2\ b$. The relationships are shown in Figure 2.

To get the data into this form, first subtract off the overscan from the bias and dome-flat images (either with ccdproc using IRAF or any other software). Then create an average bias image and the image of the standard deviation about this average. Note the bias image has real structure in it – the bias level is not zero even after subtracting off the overscan, presumably because of charge-injection during read-out. Next subtract the average from the individual dome-flats, and create and average and standard deviation for the dome-flats. Relate the mean and standard deviation, pixel by pixel. After binning by the mean count level, these data are what are shown in Figure 2.

The error-bars are the standard deviation of the mean standard-deviation value for a given mean count level; the errors in these quantitites are tiny (down by a factor of $\sim \sqrt{5e4}$.

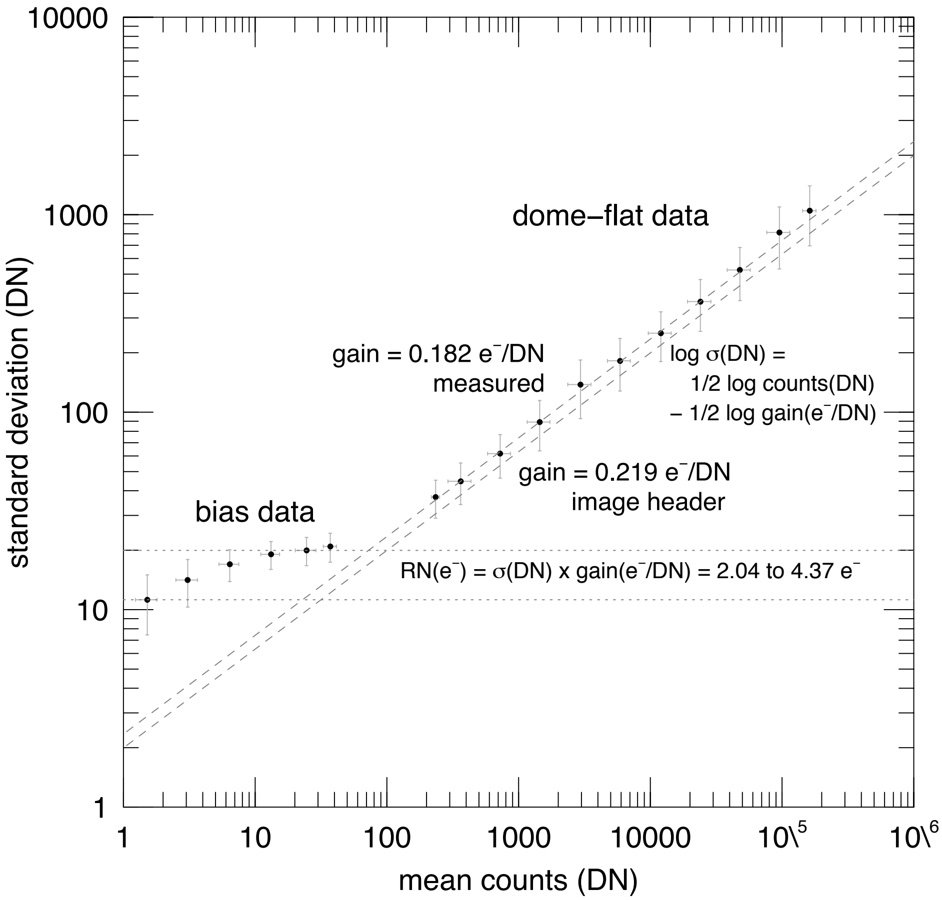

Fig. 1.— Photon propogation for Bench Spectrograph CCD STA1.