A500 / Homework 1, Spring 2017 / Solutions

All problems refer to material in Lecture 2

1. [5 points] This is hopefully a trivial excercise of dividing $f_\nu = 1$ Jy = 1e-23 erg sec^{-1} cm^{-2} Hz^{-1} by h, noting that Hz^{-1} / sec is equivalent to neper^{-1} (think: 1/sec is ν and Hz is $\Delta\nu$), and converting from cm^{-2} to m^{-2}.

2. [5 points] we have in general that $m = -2.5 \log f/f_0$. For the AB system $m_{\rm AB} = -2.5 \log f_\nu - 48.60$ which is equivalent to $f_{\nu,0} = 3631$ Jy. For the Johnson system the magnitudes are defined in the same way but $f_{\nu,0}$ is different, as given for example in Table 2.1. It follows that for a given source $(V-I)_{\rm AB} - (V-I)_{\rm Johnson} = -2.5 \log\{\ [f_{\nu,0}(V_{\rm Johnson})/f_{\nu,0}(V_{\rm AB})]\ /\ [f_{\nu,0}(I_{\rm Johnson})/f_{\nu,0}(I_{\rm AB})]\ \}$. For $f_{\nu,0}(V_{\rm Johnson}) = 3640$ Jy and $f_{\nu,0}(I_{\rm Johnson}) = 2550$ Jy, $(V-I)_{\rm AB} = 1.31$ mag.

3. [5 points] Since we are working in f_ν you don't necessarily have to work out the relation between f_λ and f_ν but it is useful to see this and it is handy for the next problem.

> The integral of the flux over a given band-pass is the same if integrated over frequency or wavelength. From this it follows that $f_\nu\ d\nu = f_\lambda\ d\lambda$. This yields $\nu f_\nu = \lambda f_\lambda$, and hence $f_\lambda \propto \lambda^{-2}$ for constant f_ν.

Returning to the immediate problem, for f_ν constant by definition $(u-R)_{\rm AB} = 0$. Let's assume u is the SDSS u' filter, which is close enough to the standard Thuan & Gunn u filter developed as an extension to the griz system of Gunn & Oke. To compute $(u-R)_{\rm Johnson}$ we need to figure out the Johnson system zeropoint for the u' filter. From "Table 8" on Slide 15 in Lecture 2 you can compute that $(u'-R)_{\rm AB} = 0.77$ mag for Vega, i.e., this source is bluer than Vega. You can adopt the R band zeropoint from Table 2.1 and solve for $f_{\nu,0}(u'_{\rm Johnson}) = f_{\nu,0}(R_{\rm Johnson}) \times {\rm dexp}(-0.4 \times 0.77)$ etc., but it is simpler to realize that given the definition of color differences in two magnitudes systems (the example for $V-I$ is given in problem #2 above) for a flat spectrum source $(u'-R)_{\rm Johnson}$ is just $-(u'-R)_{\rm AB}$ for Vega, i.e., -0.77 mag.

4. (a) [5 points] If you take the flux zeropoints in the Johnson system from Table 2.1 for H and K bands and adopt the central wavelengths from the same Table, you find that $f_\nu \propto \nu^{1.5}$ to good approximiation. On the other hand, since Vega is an A0 V star with $T_{\rm eff} = 10,000$ deg K, between 1.6 and 2.2 μm (H and K bands) Vega should be well into the Raleigh-Jeans tail of its black-body distribution where $f_\lambda \propto \lambda^{-4}$. From problem #3 it follows that $f_\nu \propto \nu^2$. Either of these answers ($\alpha = 1.5$ or 2) is adequate. Extra credit us given if you noted the discrepancy between the two results and tried to explain it. The difference is due to the fact that (i) Vega isn't perfectly characteriezd by a black-body (e.g., upper Hydrogen Brackett series is in the H bandpass), but more significantly (ii) the central wavelengths in Table 2.1 are not defined in a self-consistent way such that $\lambda_{\rm eff} = c/\nu_{\rm eff}$, as discussed on Slide 23 of Lecture 2.

(b) [5 points] To compute the $H-K$ color of Vega in the AB system all you need to do is repeat the excercise in #2 noting that $H - K$ in the Johnson system is by definition 0: $(H - K)_{\rm AB} = -2.5\log\{\ [f_{\nu,0}(H_{\rm Johnson})\ /\ f_{\nu,0}(H_{\rm AB})]\ /\ [f_{\nu,0}(K_{\rm Johnson})\ /\ f_{\nu,0}(K_{\rm AB}]\ \} = -0.52$ mag, i.e., Vega is "blue" in this color even though it appears "red" in the AB system for $(u' - R)$.

5. (a) [5 points] $S_{10} \equiv$ equivalent number of 10th mag stars deg^{-2}. An $I = 10$ mag A0 star has a flux $f_\nu = 0.255$ Jy, and 1 deg = 3600^2 arcsec^2, so $S_{10}(I) = 0.0197$ μJy arcsec^{-2}.

(b) [5 points] From #2 and the lecture notes we have $\mu_{{\rm sky},V} = 21.5$ mag arcsec^{-2} so $\mu_{{\rm sky},I} = 19.8$ mag arcsec^{-2}, both in the Johnsin system. Using the zeropoint in Table 2.1 this is equivalent to $\sim$28 μJy arcsec^{-2} or 1421 S_{10}.

6. [5 points] Since we are calculating photons per arcsec^2 and we are given a surface-brightness in mag arcsec^{-2}, we can ignore solid angle in the calculation, but we will note it for completeness. For $\mu_{rmsky,I} = 19.8$ mag arcsec^{-2} we just calculated this corresponds to 28 μJy arcsec^{-2}. We know 1 μJy = 15.1 photons sec^{-1}m^{-2} neper^{-1}. Multiplying f_ν by the band-width in nepers of 0.19 and the telescope collecting area of $\pi \times (3.50/2)^2 = 9.62$ m^2 yields $f_\gamma = 28 \times 15.1 \times 9.62 \times 0.19 = 739$ photons s^{-1} arcsec^{-2}. The question actually asked for the photon flux at the top of the atmosphere. The sky is much darker from space, particularly in the red and near-IR where atmospheric air-glow is strong. Above the atmosphere $\mu_{{\rm sky},I}$ is closer to 22.2 mag arcsec^{-2} in which case the photon flux is closer to 81 photons s^{-1} arcsec^{-2}.

7. [5 points] At 790 nm (I band), $L_{\nu,\odot} = 6.86 \times 10^{18}$ erg sec^{-1} Hz^{-1}. Since 1 pc = 3.086×10^{18} cm, 1 Jy = 9.52×10^{13} erg sec^{-1} pc^{-2} Hz^{-1}, so that $L_{\nu,\odot}$ pc^{-2} = 72033 Jy. Converting this to a surface-brightness over 4π sterad gives $L_{\nu,\odot}$ pc^{-2} sterad^{-1} = 5732 Jy sterad^{-1} = 0.134 μJy arcsec^{-2} (1 arcsec = 206265^{-1} radians). Using the result from problem #5 we have: $\mu_{sky,I} = 19.8$ mag arcsec^{-2} $\sim$ 28 μJy arcsec^{-2} = 209 $L_\odot$ pc^{-2}.