A comparative Study of Velocity Statistics of Hydrodynamic and Magnetohydrodynamic Turbulence

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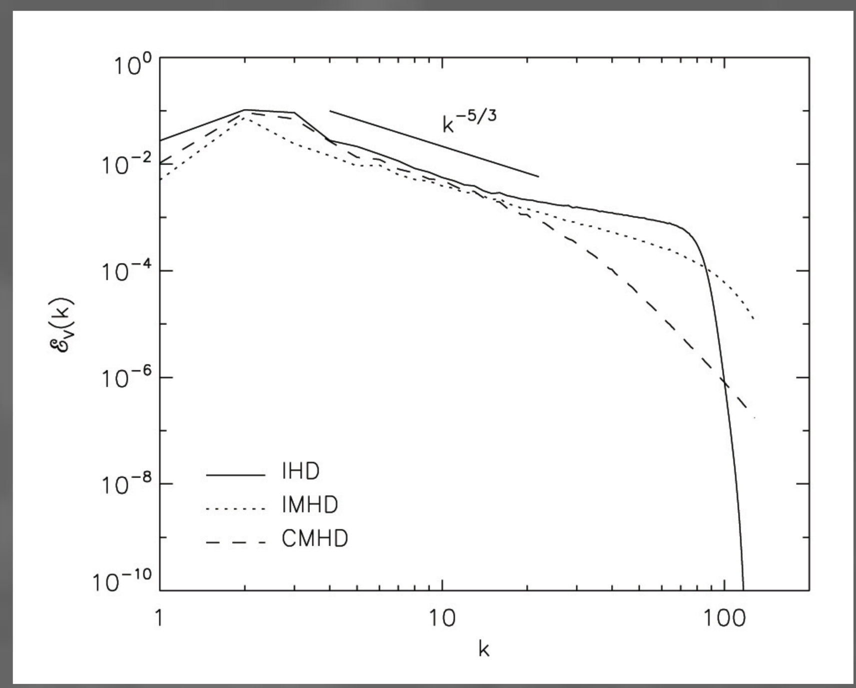
Introduction:

- Turbulence is important for many astrophysical processes such as star formation, transport of heat, and cosmic ray propagation. For astrophysical media it is important to address the differences that magnetic fields and compressibility produce. A comparative study is performed of changes that arise in the velocity statistics, or turbulent motions, as a result of individually including an external magnetic field and compressibility.
- 3 models of turbulence used:
  1. Incompressible hydrodynamic (IHD)
  2. Incompressible magnetohydrodynamic (IMHD)
  3. Compressible magnetohydrodynamic (CMHD)

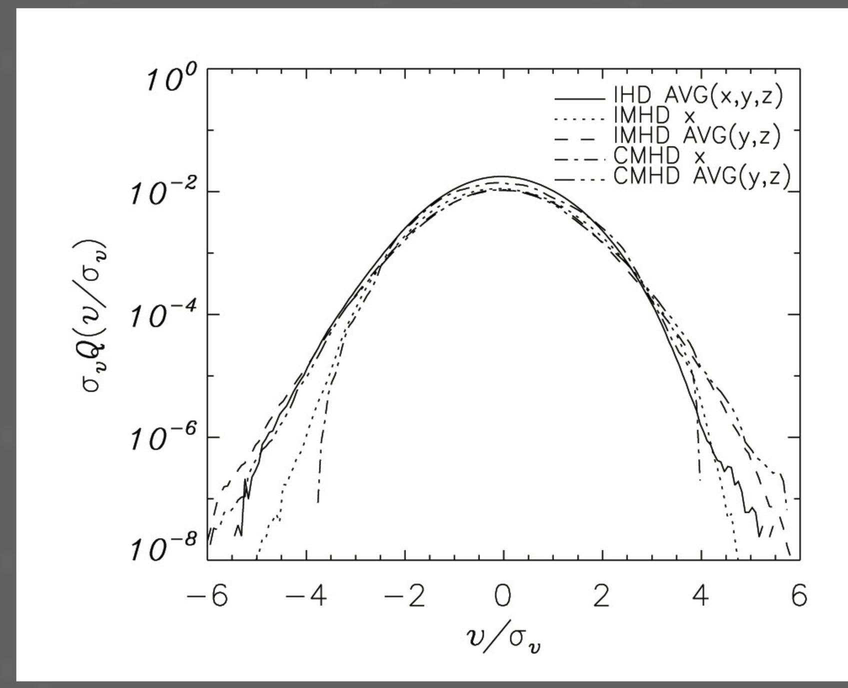
Kinetic Energy Spectra:

- Turbulence is excited then allowed to dissipate
- Inertial Range - for IHD $E(k) \propto k^{-5/3}$ (K41)
- $k \sim 1/\ell$, ℓ = separation between 2 points
- Compatibility of spectra in inertial range
- Energy transfer process is consistent



Probability Distribution Function (PDF):

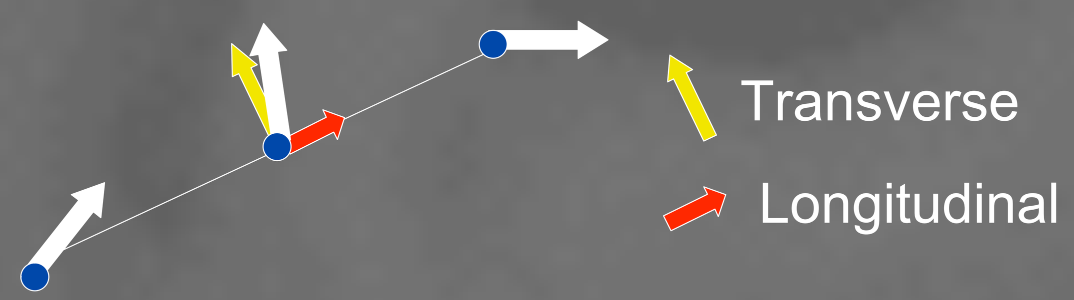
- Histogram of v-field values at each point



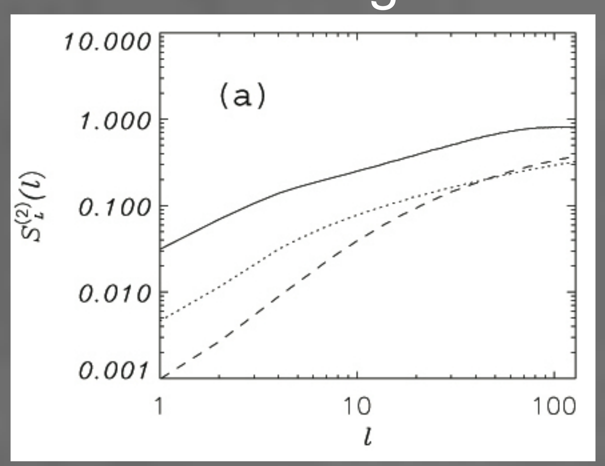
- IMHD: No sources, sinks, or shocks
- CMHD: Shocks at high v's dissipate E
  – Smallest range of v's
- MHD models: B => Alfven waves with perturbation ⊥ to B => $v_\perp$ contribution
  – PDF's in plane perpendicular to B field are like IHD while those parallel are not

Velocity Structure Functions (Global Frame):

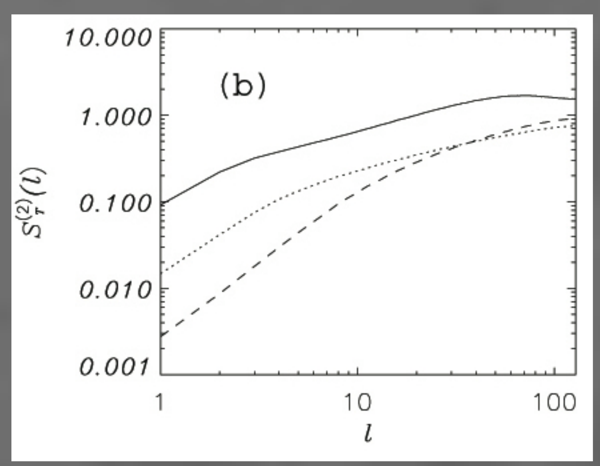
- Magnitude of difference between velocity vectors at 2 points separated by distance ℓ in 3D
- Raised to a power (order of structure function)
- Average over random points for each ℓ



2nd order longitudinal



2nd order transverse

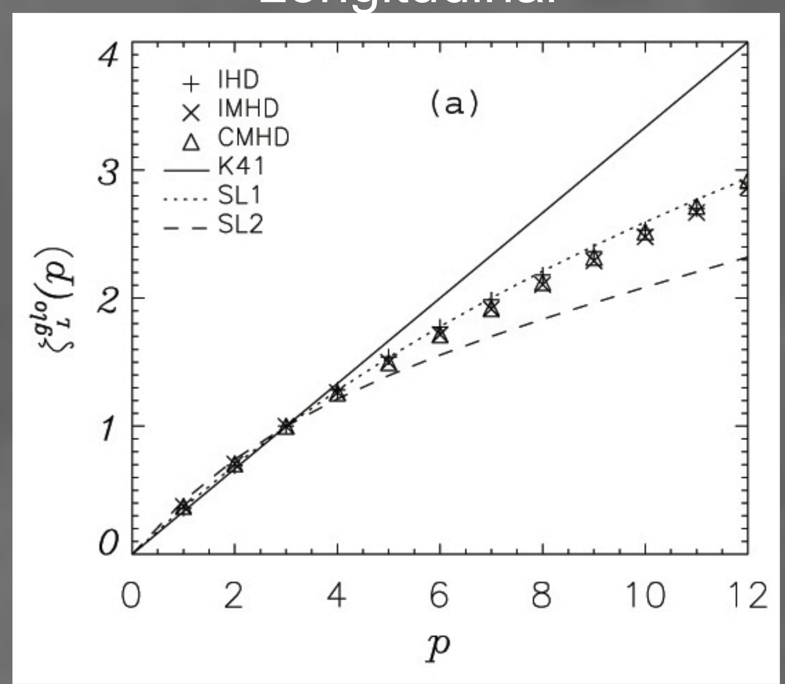


- Maximum ~ size of largest structure (radius of eddie)
- Power law dependence in the inertial range
- Can plot scaling exponents as a function of the order of structure functions

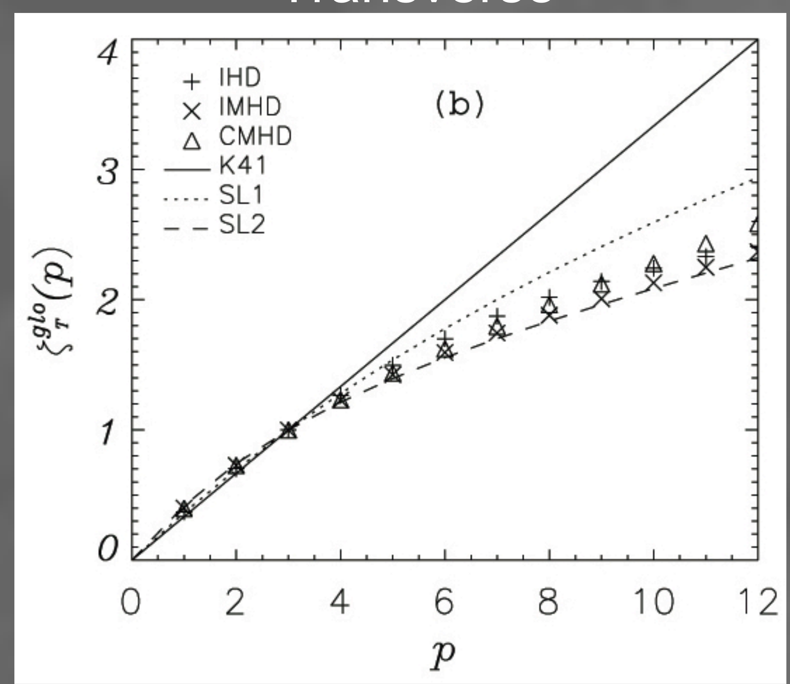
Scaling Exponents (Global Frame):

- Scale-invariance (self-similarity) = turbulence at larger scales can be reproduced by magnification of smaller scales
- Scaling exponents linear => scale invariant
- Not linear => scale-variant (intermittent)
- Dimensions of structures that dissipate the energy:
  – SL1: 1-D structures (filaments)
  – SL2: 2-D structures (sheets)

Longitudinal



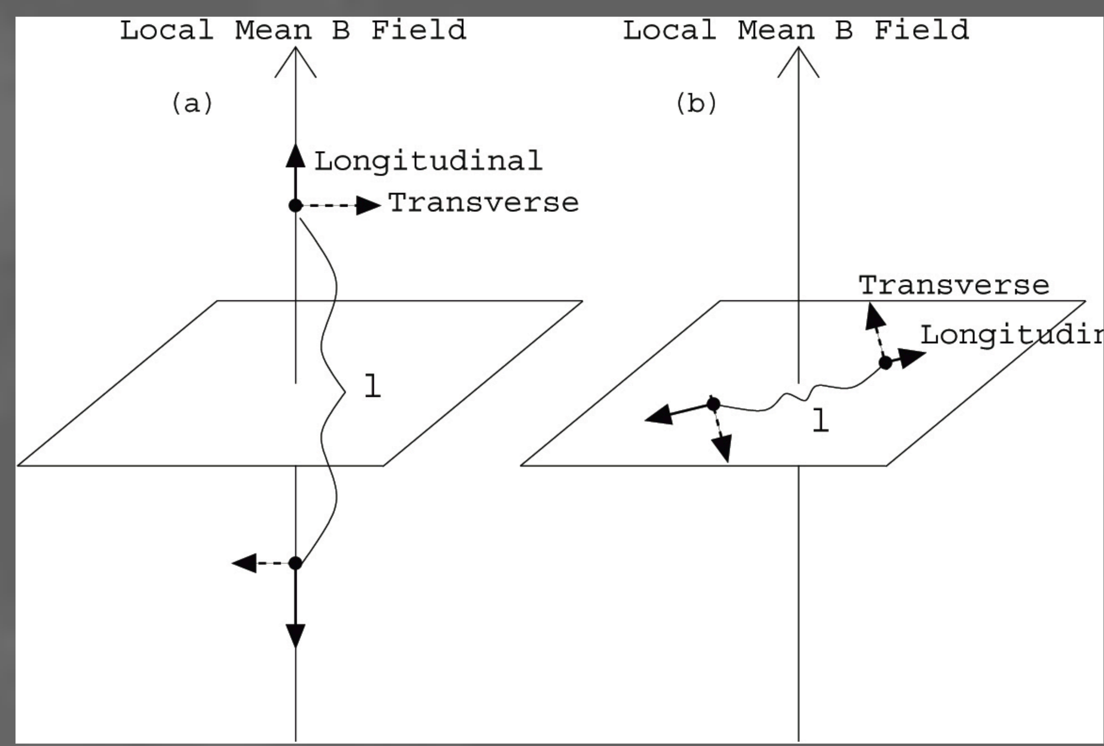
Transverse



1. High order longitudinal statistics in the global reference frame are affected very little by the inclusion of a magnetic field and compressibility.

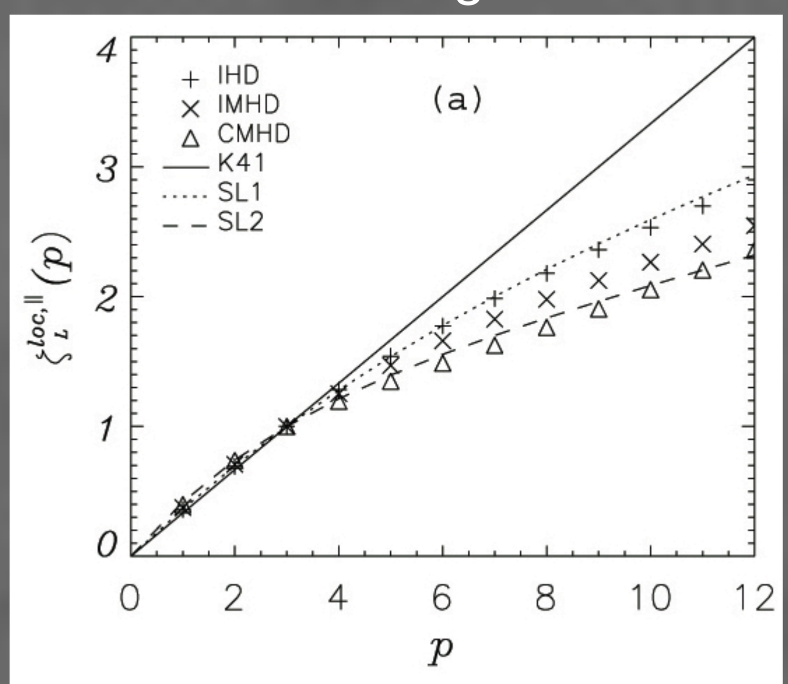
Local Reference Frame:

- Local parallel - probe along local mean B
- Local ⊥ - probe plane ⊥ to local mean B

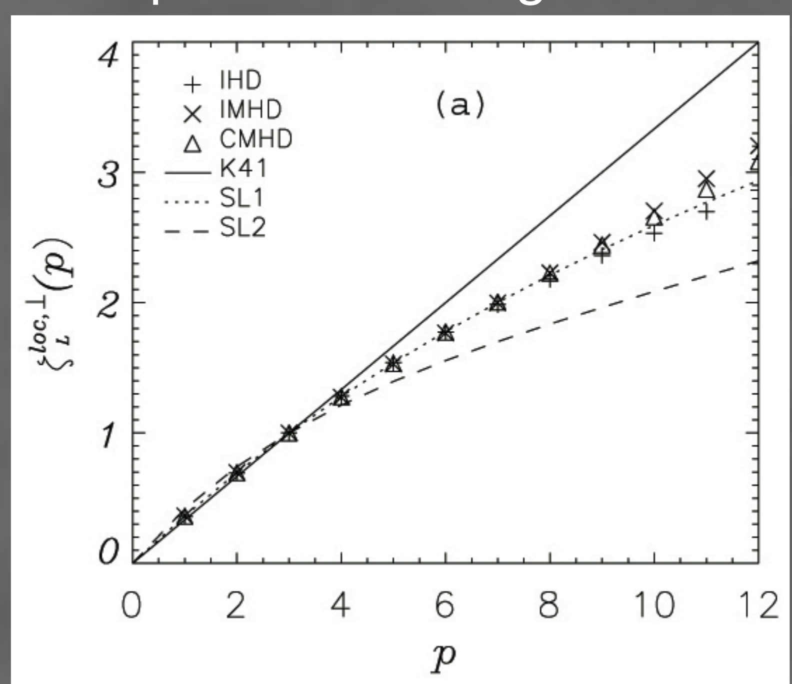


Local Frame Scaling Exponents:

Parallel Longitudinal



Perpendicular Longitudinal

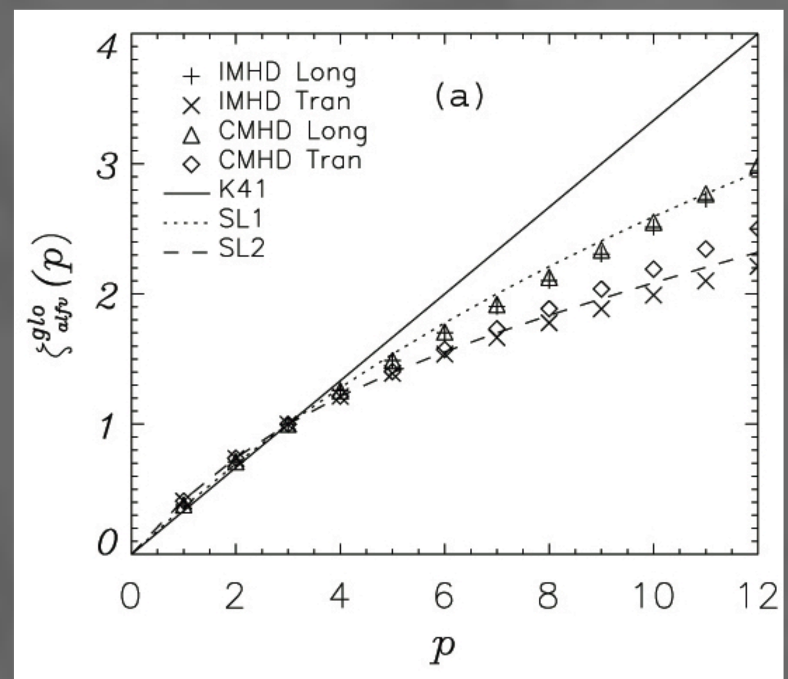


2. Turbulent motions in the plane perpendicular to the local mean field are affected very little by the inclusion of an external magnetic field and compressibility, while those parallel are affected significantly.

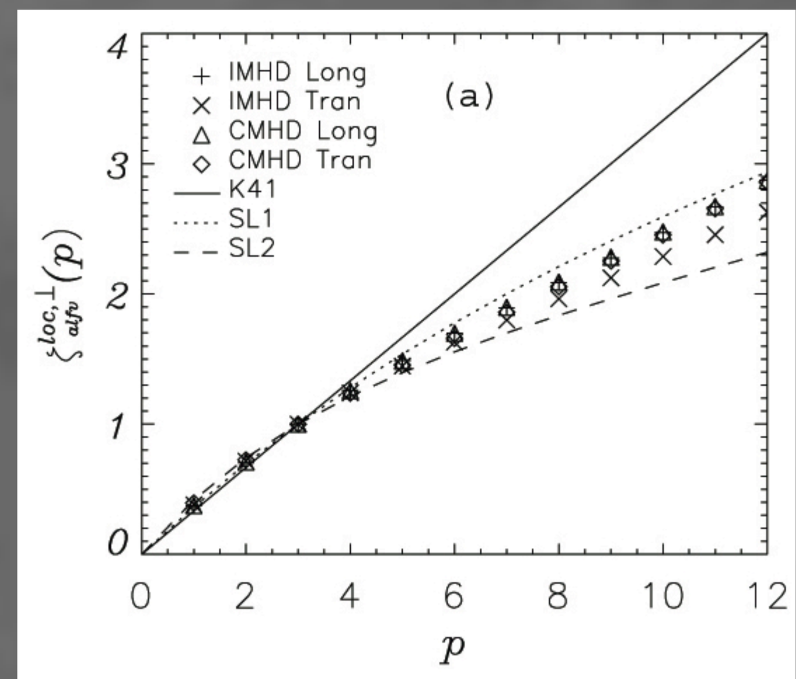
Decomposed Scaling Exponents:

- 3 types of modes or waves: Alfven, fast, and slow

Alfvenic Mode in Global Frame



Alfvenic mode in Local ⊥ frame



3. The Alfvenic mode is mostly responsible for the fact that longitudinal statistics in the global frame and motions in the plane ⊥ to the local mean field are extremely similar for all 3 models.

Conclusion:

The conclusions of this work are surprising given that the inclusion of a magnetic field and compressibility produce significant changes in the properties of turbulence. These results may be useful for astronomical observations that probe the specific components of velocity statistics considered here. This work may also help to clarify the extent to which magnetic fields and compressibility are important for different astrophysical processes.

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