

Astro 730: Homework 3 -- Comments/Solutions

1.
 - a. At apo- or perigalacticon $dR/dt=U=0$ but we have $U=-10$ km/s so the Sun is moving inwards
 - b. Note that $x=X\cos(\kappa t)$ so $v_x=-X\kappa\sin(\kappa t)$ and use this with $v_y=2(B-A)X\cos(\kappa t)$; ratio gives $\tan(\kappa t)\sim 1.4$. From velocities know that we are in the 3rd quadrant so $\kappa t\sim 234$ deg. We are approaching perigalacticon now and were at peri about 0.9 of an epicyclic P_e in the past. Using $\kappa\sim 1.4\Omega$ I get $P_e\sim 160$ Myr so we last passed peri at about 130-140 Myr in the past. This is very rough as we've, for example, ignored the effects of spiral arms and depend on poorly known dynamical quantities. The amplitude then is ~ 400 pc and we are almost at minimum radius now.
 - c. Calculate the mean density of the disk and see that whether one uses the centrl or mean density, they are close to 0.1 Msun/pc³. This then is a constant density system where $P\sim(\pi/G\rho)^{0.5}$ which gives something like 30 Myr to within a factor of 2. The point is that the z velocity is linked to density of the disk, not its orbit structure and so the relationship of v_z to velocities in the plane of the disk is subtle at best.

2.
 - a. Freeman law is $\mu(0)\approx 21.5$ B mag/arcsec² corresponding to about 140 Lsun per sq pc. Exponential disk, $L_D=2\pi h_r^2 I(0)$ or about $8E9$ Lsun, so then for the bulge $L_{bul}(B)\sim 1.2E10$ Lsun. Its approximately equal with 60% from the non-disk components.
 - b. I assume blue M/L for the younger disk of 2 and older bulge of 5 giving $M_D\approx 2E10$ Msun and $M_{bulg}\approx 6E10$ Msun. Since the mass of the ISM is \sim fewE9Msun we ignore it in this problem. For $V\sim 230$ km/s the maximum mass would be $M_{dyn}\approx 3E11$ Msun.
 - c. So we see that *at most* $\sim 25\%$ of the MW is in the form of baryonic matter. At 25 kpc we are out of baryons but $V(R)$ still remains flat.
 - d. Assume isotropic orbits(yipes! But what else can we do in a problem?), then $M\sim 3\sigma^2 R/G=4E11$ Msun. Maximum luminous fraction = 18%. Mean density is a whopping $1E-4$ Msun/pc³.

3.
 - a. If we have Freeman's law pure disks, then $L\propto M=2\pi h_r^2 \Sigma(0)=V^2 R/G$. We then have $V^4\sim L\Sigma(0)*\text{constants}$ --thus we get the TF relationship. (However, we still are in a physics-free zone as we haven't explained Freeman's "law" which, as we discussed in class is actual only a trend. [Extragalactic astronomy is an easy place to go into the astrophysical "law" business!])
 - b. (Note typo in the problem!) We get a too low velocity--right slope, wrong zero point for a pure disk. We'd get a velocity of about 100 km/s at $M_b=-19$, vs. ~ 140 km/s from Schneider Figure 3.19. No too far off, but it just gets worse as R increases as baryons become less important. This closeness is related to the rotation curve matching issue that we discussed in class.

- c. See eqs. 3.16 and 3.17 in Schneider.
- 4.
- a. As L increases, intensities decrease, so stellar densities must go down.
- b. Both dispersions and sizes increase as can be easily calculated from the virial theorem (a favorite prelim question, by the way). Thus density decreases. Hmm, this is consistent with the trend among luminous E galaxies so they might have formed through interactions of stellar systems (why stellar systems and not gas-rich objects?).
- 5.
- a. You need to solve $\rho(r) = (4\pi G r^2)^{-1} d/dr[r^2(d\Phi/dr)]$ which is Poisson's eq for a spherical system.

$$\rho(r) = \frac{3GMr^2 a_p^2}{(r^2 + a_p^2)^{5/2}}$$

5. Secret here is to recognize that near the center we can take the path length of the integral to get surface density to be \sim constant, so $\rho(r) \propto \Sigma(r)$. Substitution then gives $r_c \sim 0.6a_p$.
- 6.
- a. Faint end of LF suffers from every bias you can think of: surface brightness selection, flux selection, statistical distribution of galaxies since we are generally stuck with nearby samples. At the faint end the Schechter function is roughly a power law, $\Phi(L)dL \sim \text{const}(L/L_*)^\alpha(dL/L_*)$. If we then require that the integral of $\Phi(L)$ over L converge we get $\alpha > -1$. but of course we don't expect this function to extend over all L ...
- b. The product $L\Phi(L)$ peaks near L_* and thus these galaxies contribute most of the luminosity. You can easily calculate this; see also Sparke & Gallagher Fig. 1.16 in the 2nd edition.
- c. From Schneider Fig. 3.22, we get $\sigma \sim 120$ km/s. Well, it seems that for dispersions much larger than this we go from galaxies to systems of galaxies. The steep drop in number of galaxies above L_* evidently reflects a change in the nature of bound systems away from stellar galaxies.