

1.
 - a. Collisionless Boltzmann—particles moving in smooth potential, particles neither created nor destroyed, energy of system conserved so no dissipation in interactions, particle-particle interactions unimportant. Usually also applied when system has reached near steady state so system is time-independent.
 - b. Collisionless Boltzmann model implies structure of 6-dimensional phase space. Studies require knowing (x,y,z) of star thus distance and location in 3-space as well as all 3 velocity components. This kind of data are only available for Galactic stars, and currently with any precision only for Galactic stars with $D < \text{few hundreds of pc}$.
 - c. In the quasi-ergodic case stars locations will eventually cover most of the area available on an energy hypersurface. This is NOT what's seen; instead orbits are semi-regular indicating that additional factors (e.g. “third integrals”) bound the available phase space for a given orbit.
 - d. Yes, with more constraints one can better represent the distribution of orbits in the Milky Way with a finite number of orbit families. If on the other hand the orbits were quasi-ergodic then this would be a poor approximation—a few orbit families wouldn't even come close to representing the true structure of the system.

2.
 - a. The difference of $A-B=V_0/R_0$ and using $R_0=8$ kpc we get $V_0=220$ km/s. Note that the errors in Oort's A and B combine with errors in the distance to make V_0 depressingly uncertain so you will see a variety of values used in the literature.
 - b. We can derive $V(R)$ for $R < R_0$ using the tangent point method and assuming HI is in on circular orbits. Since the HI line is at 21 cm it is not affected by dust so we can “see” the inner Galaxy. Optical depth in the line is also a relatively minor concern. The problems with $R < 3$ kpc are a lack of gas disk tracers and the presence of a bar that produces strongly non-circular orbits, the x_1 and x_2 orbit families. If we use stars as tracers then extinction and velocity dispersion also become factors. Remember that we cannot go to arbitrarily long IR wavelengths and still detect the sharp absorption lines from stellar photospheres, thus we can't always manage to beat the effects of extinction and still measure stellar radial velocities.

For $R > R_0$ we need to have tracers with measured distances, and this means stellar standard candles. Extinction and standard candle identification become twin problems. Examples of objects that can be used for this are star clusters or Cepheid variable stars. As a result the outer rotation curve is not particularly well known.

- c. Limitations are we assume a spherical mass distribution and are not sensitive to mass beyond where V is determined. $M(R)$ diverges linearly with R while the baryonic mass (gas+stars) converges, so there is mass that is not being included in the census of luminous material.

- d. The idea here is to apply a version of the collisionless Boltzmann equation to relate the stellar density and scale height self consistently to the observed velocity dispersion in the z direction.

$$2\pi\Sigma(< z)G \approx -\frac{1}{n(z)} \frac{d}{dz}(n(z)\sigma_z^2)$$

Thus we see that if $(n(z)\sigma_z^2)$ declines rapidly, then we have a high mass surface density.

If there are two self-gravitating disks embedded within one another, then at small scale heights you can measure mainly properties of the thin disk with its lower dispersion and higher density. At large z the second disk will increasingly dominate. This will lead to overestimates of both number densities of stars and dispersion relative to the pure thin disk. As a first approximation lets take σ_z^2 to be constant for both disks, but about a factor of 4 higher in a thick disk. Example for exponential case

$$\frac{d}{dz}(n(z)\sigma_z^2) \sim -\frac{\sigma^2}{h_z} n_0 \exp(-z/h_z)$$

Thus if we assume thick disk stars are in the thin disk, we may make the scale height slightly too large, and then the exponential and squared velocity dispersion terms win and we overestimate the thin disk mass density.

Suppose the HI had a much higher mass density than we thought. It is a very thin component of the Galaxy, so you could think of it as a dense sheet of mass in the Galactic midplane. You would then calculate a much smaller z-scale height for a given σ_z than if the system only contained visible stars and gas. On the other hand, since the HI surface density tends to trace the dark matter density, putting extra mass in the HI would then yield a proper flat rotation curve.

3. Drawings of the Milky Way should include--Side view: nucleus, bulge, thin & thick disks, halo or stellar spheroid, DM halo + possibly warp, globular clusters. Top view: nucleus, bar, spiral arms, + possibly ring, star forming regions.
4. See comments on your answers--most types okay but many tended to classify spiral galaxies as one step too early; e.g. an Sb being called an Sa, etc. I think this is a result of comparing multi-wavelength images with blue light photographic standards. In blue light the spiral arms stand out more (why?).
 - a. Clusters of galaxies are dominated by early type galaxies, E & S0 classes, with few spirals. This is thought to be the result of environmental processes including ram pressure stripping of gas and tidal disruption/thickening of disks [+ some things we don't understand]. NGC1129 hints at another environmental effect in dense regions as its much larger than any of its companions. We'll discuss this later in the context of "galactic cannibalism".

- b. These galaxy systems, such as Seyfert's sextet, are compact groups where multiple interactions are occurring between member galaxies. Key point is that low velocity interactions are very disruptive and as several pointed out, mergers of disks can lead to E galaxies.
- c. E and S0 galaxies: the E is on average denser in its central region but as both galaxies have roughly the same number of stars and these are spread over a larger volume in the 3D E, its overall density can be lower. Since both are "dead red" classes of galaxies, all stars are old and thus stellar masses are likely to be similar.

And there is no obvious way to make an S0 from 2Es. The Es are spheroidal systems and you can't take what is effectively high entropy E galaxies and convert them to lower entropy organized motions in a disk with a lower dispersion via an interaction that adds energy to the system. BUT you can make disks into spheroids by increasing their velocity dispersions in interactions.