Primitive Model of Turbulent Viscosity

Ellen Zweibel

zweibel@astro.wisc.edu

Departments of Astronomy & Physics

University of Wisconsin, Madison

and

Center for Magnetic Self-Organization

in

Laboratory and Astrophysical Plasmas

Overarching Question

How to incorporate a tangled magnetic field into astrophysical gas dynamics?
This impacts the structure & stability of

- self gravitating clouds
- disks
- ICM
- shocks

Traditional approaches based on turbulent pressure. I'll include dissipation.

The Plan of This Talk

- Turbulence model of Ryutov & Remington
- Dynamical equations with this turbulence model
  - Dissipation leads to an effective viscosity
- Examples
  - Waves
  - Rayleigh-Taylor instability

Turbulence Model

- Irregular magnetic field B on scale $\leq l$.
- B relaxes to its minimum energy state (consistent with constraints such as global helicity) on timescale τ.
- Response to a flow $\delta\mathbf{V}$:

$$\frac{\partial\delta\mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) - \frac{\delta\mathbf{B}}{\tau}.$$

- We'll assume $\delta\mathbf{V}$ varies on a scale $L \gg l$

Turbulent Stress

- The perturbation $\delta\mathbf{B}$ induces

$$\delta\mathbf{F} = -\nabla \cdot \delta\mathbf{T}$$

- Perturbed stress tensor

$$4\pi\delta T_{ij} = \delta_{ij}\mathbf{B} \cdot \delta\mathbf{B} - B_i\delta B_j - B_j\delta B_i$$

- Find turbulent stress on large scale L by averaging over l.

Homogeneous, Isotropic Turbulence

$$\langle B_i B_j \rangle = \frac{1}{3} \langle B^2 \rangle \delta_{ij}$$

$$\left(\frac{\partial}{\partial t} + \frac{1}{\tau} \right) \langle \delta T_{ij} \rangle = -\frac{1}{3} \frac{\langle B^2 \rangle}{4\pi} \left(\frac{\partial \delta V_i}{\partial x_j} + \frac{\partial \delta V_j}{\partial x_i} \right)$$

Waves with HIT

When $\tau \to \infty$, there are 2 wave modes:

- Shear waves: $\omega^2 = k^2 v_A^2/3$
- Compressive waves: $\omega^2 = 2k^2 v_A^2/3$

Compare to a uniform field

- Shear waves: $\omega^2 = (\mathbf{k} \cdot \mathbf{v}_A)^2$
- Compressive waves: $\omega^2 = k^2 v_A^2$

Viscosity from HIT

When $\tau \to 0$,

$$\langle \delta T_{ij} \rangle = -\frac{\tau v_A^2}{3\rho} \left(\frac{\partial \delta V_i}{\partial x_j} + \frac{\partial \delta V_j}{\partial x_i} \right)$$

Coefficient of kinematic viscosity ν_T

$$\nu_T \equiv \frac{1}{3} v_A^2 \tau$$

Magnitude of Viscosity

Compare magnetic & plasma viscosities:

$$\frac{\nu_T}{\nu_p} \sim \frac{v_A^2 \tau}{v_i^2 \tau_i}$$

Rayleigh-Taylor Instability

Classic setup: sharp interface separating ρ_2 from ρ_1 below. Ripple interface as e^{nt+ikx}. With no B or ν,

$$n^2 = \gamma_{RT}^2 \equiv kg\frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$$

- Laminar magnetic field $\hat{x}B$ stabilizes perturbations with $k > g\Delta\rho/B^2$, B_y has no effect.
- With viscosity, fastest growth rate is at $k \sim (g/\nu)^{1/3}$

Add Turbulence

Assume 2D turbulence is present, no B threading the interface. Derive the dispersion relation

$$n^2(n\tau + 1) + n\tau(\gamma_A^2 - \gamma_{RT}^2) - \gamma_{RT}^2 = 0$$

where $\gamma_A^2 \equiv k^2 B^2/(4\pi(\rho_2 + \rho_1))$.

- Without relaxation, modes with $\gamma_A^2 - \gamma_{RT}^2$ are stable (no directional effect)
- With relaxation, $n_1 n_2 n_3 = \gamma_{RT}^2/\tau$: instability for $\rho_2 > \rho_1$.

Summary

- The RR model of turbulence invokes scale independent relaxation on a timescale τ
- Included this in a standard derivation of averaged magnetic stress.
- Without relaxation, isotropic turbulence supports shear & compressive waves; 2D turbulence stabilizes RT instability at small enough scales.
- Relaxation introduces a viscous response with viscosity $\nu_T = v_A^2\tau/3$ & destabilizes RT modes, similar to classical viscosity.
- If this is sound, there are many other applications.