Dissipation Range of MHD Turbulence

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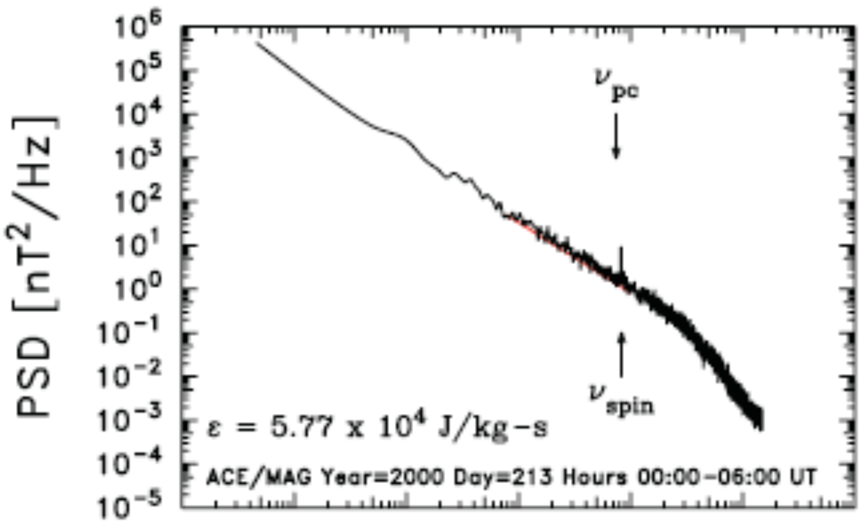
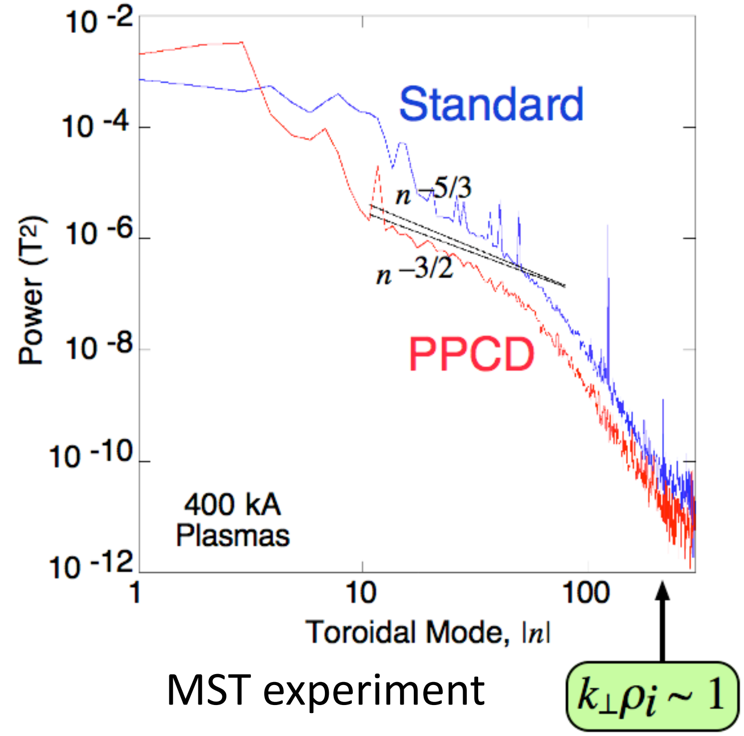
Magnetic fluctuation spectra in lab and astrophysical plasmas show "dissipation range"



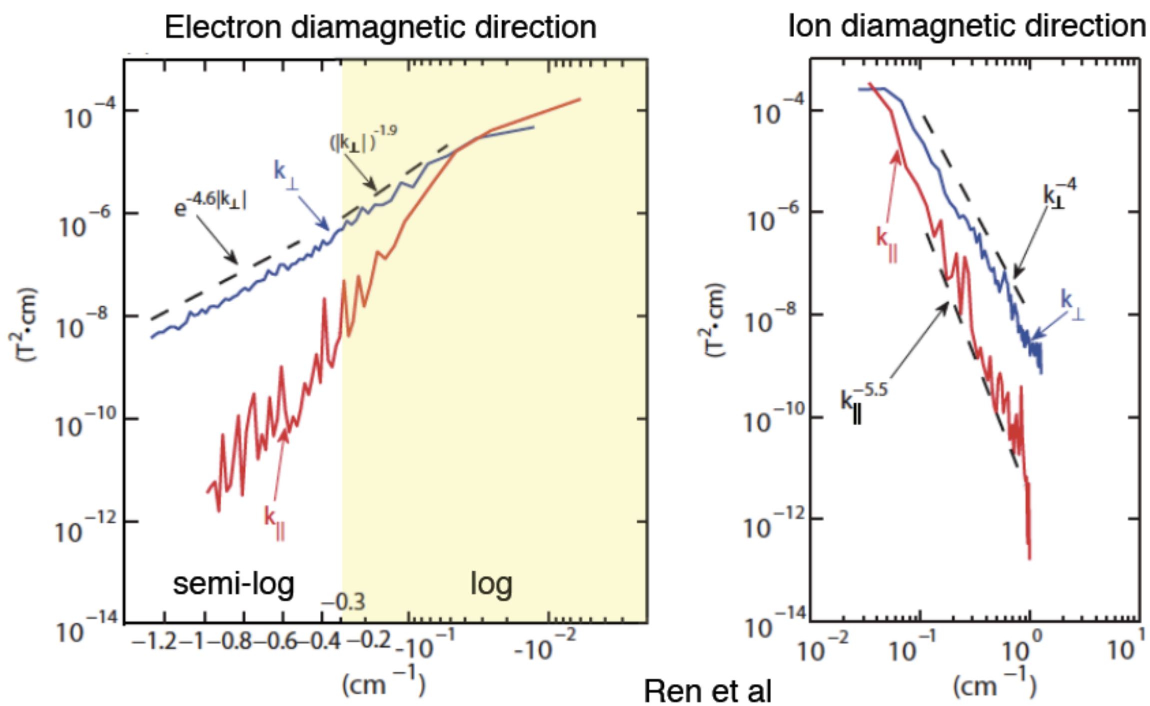
Solar wind from Smith et al.



MST experiment

Steeper falloff: transition to kinetic Alfvén wave regime? classical dissipation (resistive, viscous)? kinetic dissipation?

MST spectrum: anisotropy, power law, and exponential decay



What is the physics of the exponential decay?

Is it related to characteristic exponential spectra in MHD?

What are characteristic exponential spectra in MHD?

Is there a relation to heating processes (e.g., coronal heating)?

Derive dissipation-range spectra for MHD under collisional damping from resistivity and isotropic viscosity

Compare with experiment: exponential decay from resistivity, viscosity?

First develop theory for simple isotropic viscosity and resistivity:

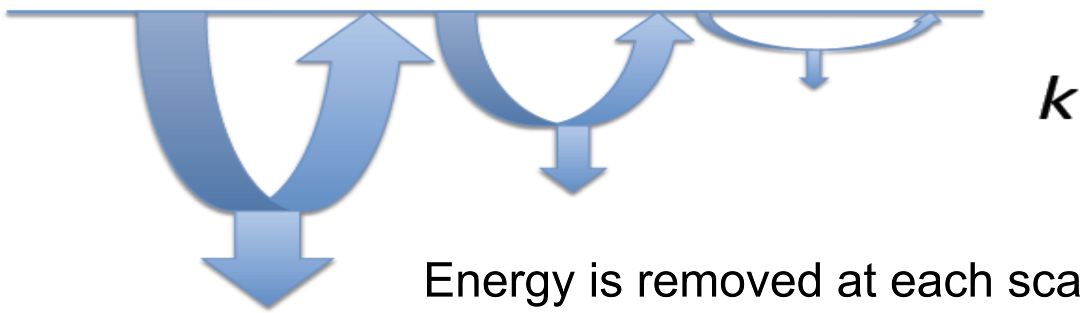
Carry over closure techniques from hydrodynamics, focusing on issues in MHD that do not occur in hydrodynamics

- Alfvénicity
- Anisotropy
- Alignment
- Nonlocal cascading
- Magnetic Prandtl number

From standpoint of dissipation range spectra, anisotropic viscosity, kinetic dissipation are harder problems

Initial results last year; complete analysis this year

Spectrum set by relative rates of scale-dependent nonlinear transfer and dissipation



Energy is removed at each scale

Hydrodynamics: nonlinearity keeps transferring energy (self similar across scales) as it is removed by dissipation

Dissipation attenuates energy transfer rate in wavenumber space:

Pm governs differential dissipation rate

$$-2\eta k^2 \mathrm{E}_B(k) = d\mathrm{T}_B(k)/dk$$

$$-2\nu k^2 \mathrm{E}_V(k) = d\mathrm{T}_V(k)/dk$$

Alfvénicity, alignment, anisotropy, nonlocality reside in spectral transfer rate

To solve dissipation-range rate balances, nonlinear transfer rate correlations must be closed

Transfer rates $T_B(k) = \int [\mathbf{B}\cdot(\mathbf{v}\cdot\nabla)\mathbf{B} - \mathbf{B}\cdot(\mathbf{B}\cdot\nabla)\mathbf{v}]\exp(i\mathbf{k}\cdot\mathbf{x})d^3x$

and $T_V(k) = \int [\mathbf{v}\cdot(\mathbf{v}\cdot\nabla)\mathbf{v} - \mathbf{v}\cdot(\mathbf{B}\cdot\nabla)\mathbf{B}]\exp(i\mathbf{k}\cdot\mathbf{x})d^3x$

must be written in terms of energies

$E_B(k) = \int v^2 \exp(i\mathbf{k}\cdot\mathbf{x})d^3x$ and $E_V(k) = \int B^2 \exp(i\mathbf{k}\cdot\mathbf{x})d^3x$

In hydrodynamics, 9 closures give $\mathrm{E}(k) = \varepsilon^{2/3}k^{-5/3}\exp\left[-b\left(\frac{k}{k_d}\right)^{\alpha}\right]$

Tennekes and Lumley closure: transparent, intuitive, close to experiment

- Write T dimensionally
- Convert field appearing quadratically to to energy
- Remaining field: specify wavenumber variation from Obukov balance
- Alignment: scale-dependent factor in transfer rates, Obukov balance

Weaker nonlinearity enhances steepness of dissipative falloff

Hydrodynamics: $$\mathrm{E}(k) = \varepsilon^{2/3} k^{-5/3} \exp\left[-b\left(\frac{k}{k_d}\right)^{\alpha}\right]$$

Power of k in exponential factor determined by $-2\nu k^2\, \mathrm{E}_V(k) = d\mathrm{T}_V(k)/dk$

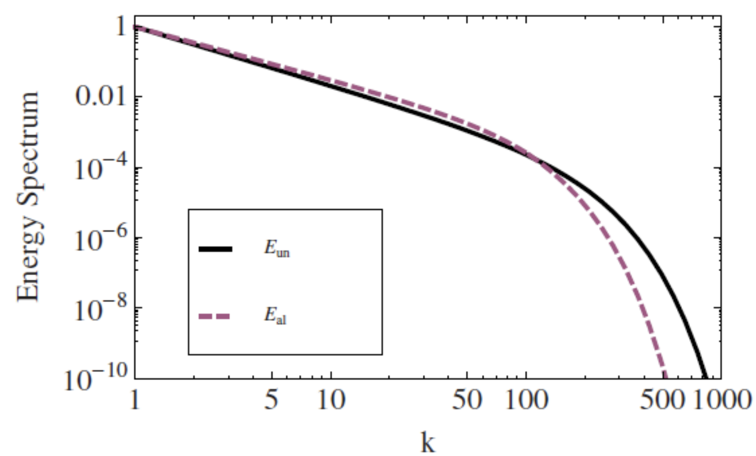
Heuristically: exponential factor set by energy dissipation in eddy turnover time

=> Alignments affects exponential decay

$$\exp\left[-b\left(\frac{k}{k_d}\right)^{\alpha}\right] \propto \exp\left[-2\nu \mathrm{k}^2 t\right]\Big|_{t=\tau_{eddy}}$$

Unaligned: nonlinearity stronger, eddy turnover time shorter, amount of energy dissipation smaller ⇒ weaker exponential falloff

Aligned: nonlinearity weaker, eddy turnover time longer, amount of energy dissipation larger ⇒ stronger exponential falloff



Closure allows calculation of dissipation range spectra for both unaligned and aligned turbulence

Pm = 1 ($\eta = \mu$):

$$-2\eta E_{\pm}(k)k^2 = \frac{dT_{\pm}}{dk}$$

$$E_{\pm}(k) = \int Z_{\pm}^2 \exp[i\mathbf{k}\cdot\mathbf{x}]d^3x$$

$$T_{\pm} = Z_{\pm}^2 Z_{\mp}\Theta_k k$$

where Θ_k is scale dependent alignment

Unaligned turbulence: $\Theta_k = 1$ for all k

$$T_{\pm} = E_{\pm}(k)\varepsilon^{1/3}k^{5/3}$$ upon closure

$$E_{\pm}(k) = a\varepsilon^{2/3}k^{-5/3}\exp\left[-\frac{3}{2}\left(\frac{k}{k_{\eta_{un}}}\right)^{4/3}\right]$$ where $$k_{\eta_{un}} = \frac{\varepsilon^{1/4}}{\eta^{3/4}}$$ is the dissipation wavenumber

Pm = 1 aligned turbulence is more complicated because scale dependent alignment only applies to inertial scales

Pm = 1 ($\eta = \mu$):

Unaligned turbulence: $\Theta_k = 1$ for all k

$$\Theta_k = \frac{\varepsilon^{1/4}}{V_A^{3/4} k^{1/4}} \quad (k < k_{\eta_{al}})$$

scale dependent alignment in inertial scales

$$\Theta_k = \frac{\varepsilon^{1/4}}{V_A^{3/4} k_{\eta_{al}}^{1/4}} \quad (k \geq k_{\eta_{al}})$$

alignment angle frozen above dissipation scale

closure yields different transfer rates above and below dissipation scale:

$$T_\pm = E_\pm(k)\varepsilon^{1/2} k^{3/2} / V_A^{1/2} \quad (k < k_{\eta_{al}})$$

$$T_\pm = E_\pm(k)\varepsilon^{1/2} k^{5/3} k_{\eta_{al}}^{-1/6} / V_A^{1/2} \quad (k \geq k_{\eta_{al}})$$

$$k_{\eta_{al}} = \frac{\varepsilon^{1/3}}{V_A^{1/3}\eta^{2/3}}$$

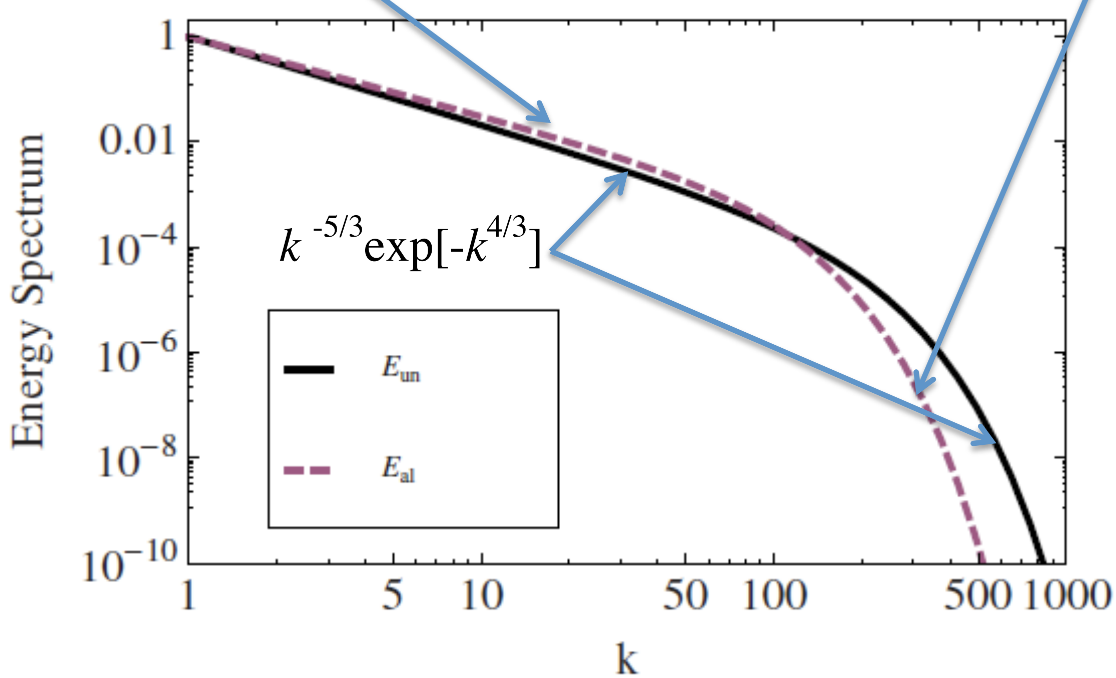
is the dissipation wavenumber

with two spectrum forms

Pm = 1 aligned spectrum has power laws and exponents characteristic of both aligned and unaligned turbulence

$$E_\pm(k) = a_< \varepsilon^{1/2} V_A^{1/2} k^{-3/2} \exp\left[-\frac{4}{3}\left(\frac{k}{k_{\eta_{al}}}\right)^{3/2}\right] \quad (k < k_{\eta_{al}})$$

$$E_\pm(k) = a_< \varepsilon^{1/2} V_A^{1/2} k^{-5/3} k_{\eta_{al}}^{1/6} e^{1/6} \exp\left[-\frac{3}{2}\left(\frac{k}{k_{\eta_{al}}}\right)^{4/3}\right] \quad (k \geq k_{\eta_{al}})$$



For Pm < 1, nonlocal interactions above resistive cutoff make spectra more complicated

Transfer rates for Pm = 1 apply to the inertial scales of Pm < 1 cascade

In dissipation range, nonlocal interactions with large scale magnetic field of inertial range allow power law for B when $k_\eta < k < k_\mu$

Unaligned turbulence:

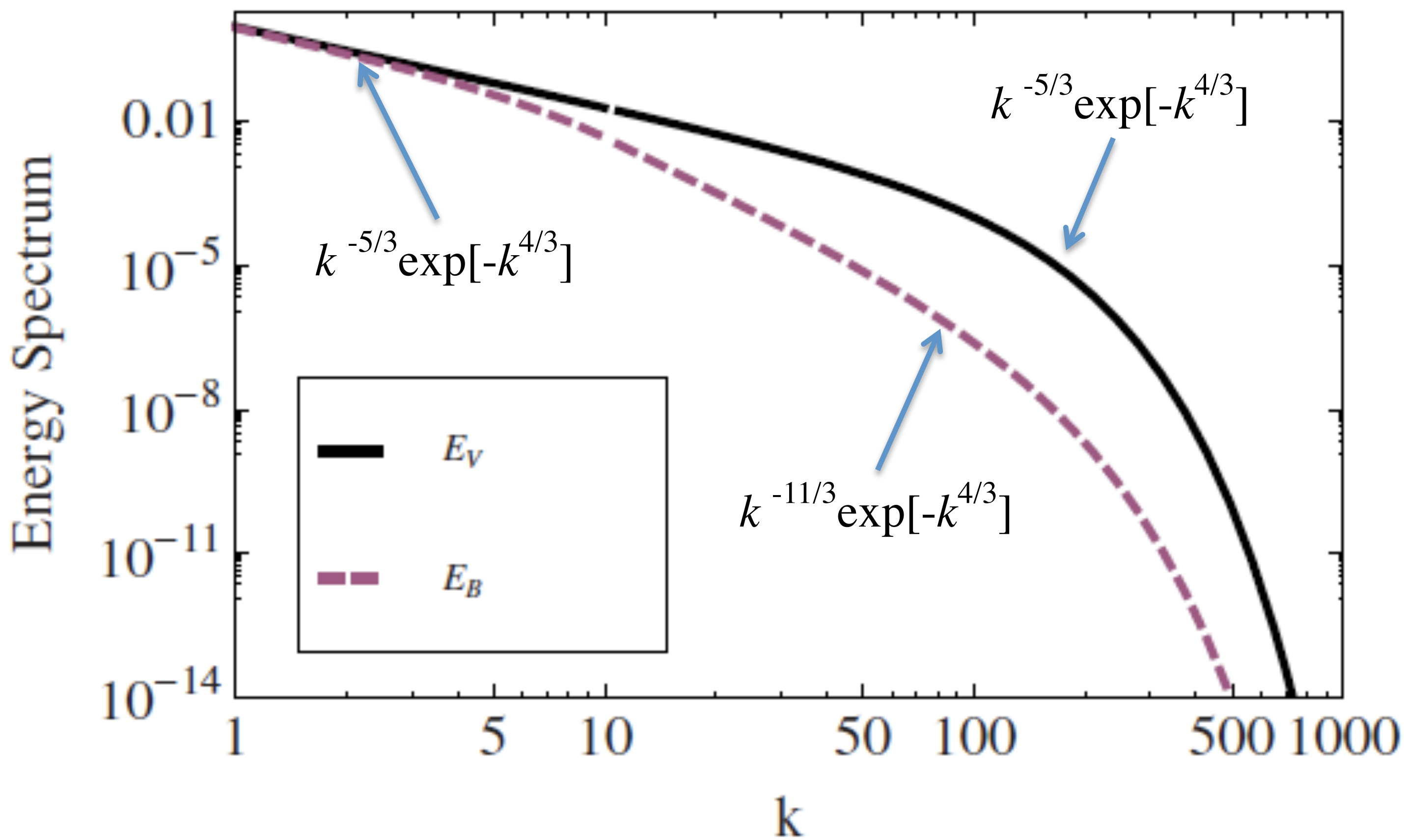
$$E_V(k) = a_< \varepsilon^{2/3} k^{-5/3} \exp\left[-\frac{3}{2}\left(\frac{k}{k_{\mu_{un}}}\right)^{4/3}\right] \quad \text{where } k_{\mu_{un}} = \frac{\varepsilon^{1/4}}{\mu^{3/4}}$$

$$E_B(k) = a_< \varepsilon^{2/3} k^{-5/3} \exp\left[-\frac{3}{2}\left(\frac{k}{k_{\eta_{un}}}\right)^{4/3}\right] \quad \text{for } k < k_{\eta_{un}}$$

$$E_B(k) = a_< \varepsilon^{2/3} k^{-11/3} k_{\eta_{un}}^2 \exp\left[-\frac{3}{2}(1-\mathrm{Pm})\right] \exp\left[-\frac{3}{2}\left(\frac{k}{k_{\eta_{un}}}\right)^{4/3}\right] \quad \text{for } k \geq k_{\eta_{un}}$$

$$\text{where } k_{\eta_{un}} = \frac{\varepsilon^{1/4}}{\eta^{3/4}}$$

Spectrum for unaligned turbulence with Pm < 1



For inertially aligned turbulence with Pm < 1, spectrum has most complicated form

v and B weakly dissipated at different rates in inertial range

Alignment lost above resistive dissipation wavenumber

Nonlocal coupling gives power law for B above resistive wavenumber

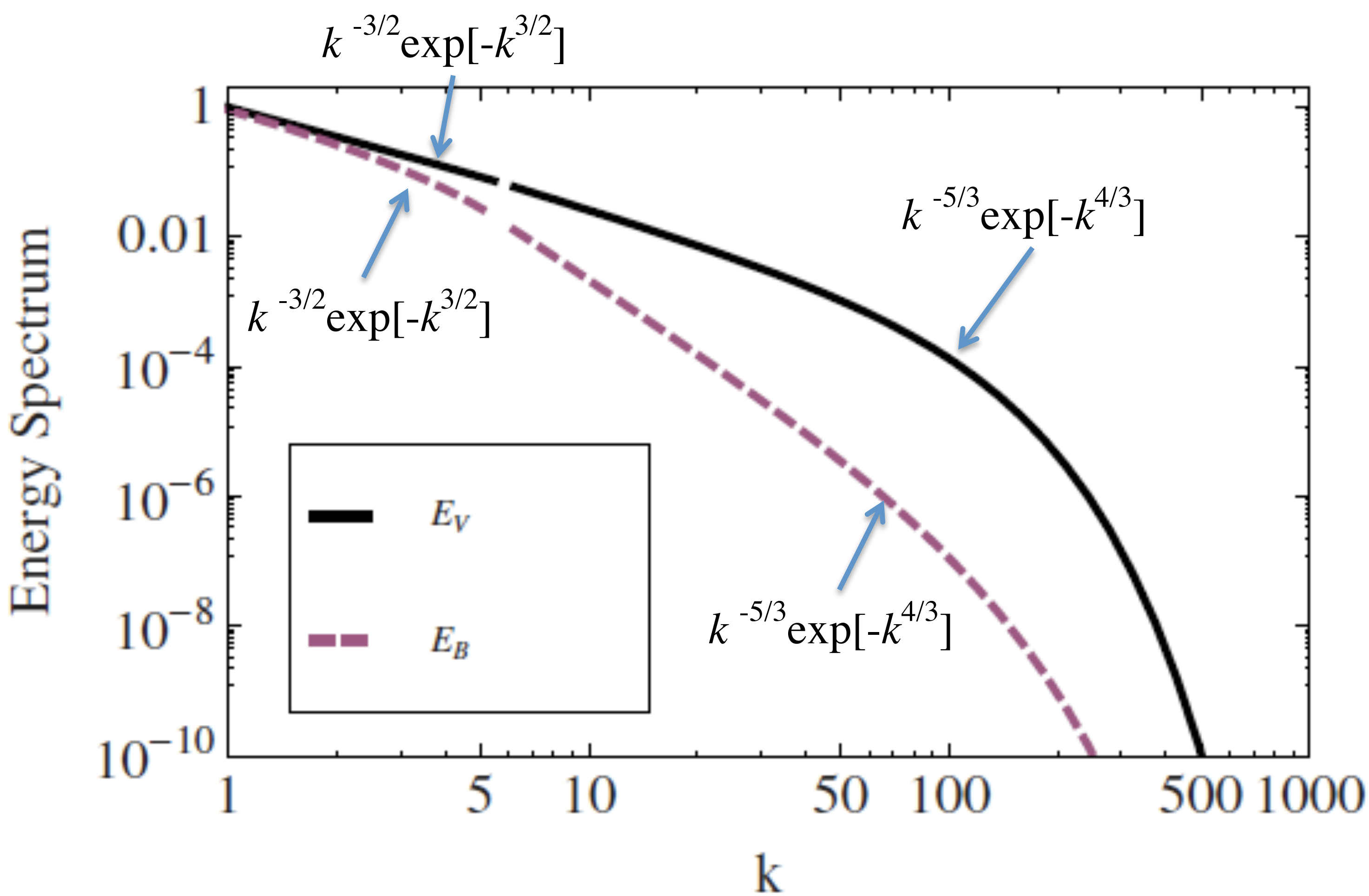
$$E_V(k) = a_< \varepsilon^{1/2} V_A^{1/2} k^{-3/2} \exp\left[-\frac{4}{3}\left(\frac{k}{k_{\mu_{al}}}\right)^{3/2} \right] \quad \text{for } k < k_{\eta_{al}}$$

$$E_B(k) = a_< \varepsilon^{1/2} V_A^{1/2} k^{-3/2} \exp\left[-\frac{4}{3}\left(\frac{k}{k_{\eta_{al}}}\right)^{3/2} \right] \quad \text{for } k < k_{\eta_{al}}$$

$$E_V(k) = a_< \varepsilon^{1/2} V_A^{1/2} k^{-5/3} k_{\eta_{al}}^{1/6} \exp\left[-\frac{4}{3}\mathrm{Pm} + \frac{3}{2}\left(\frac{k_{\eta_{al}}}{k_{\mu_{un}}}\right)^{4/3} \right] \exp\left[-\frac{3}{2}\left(\frac{k}{k_{\mu_{un}}}\right)^{4/3} \right] \quad \text{for } k \geq k_{\eta_{al}}$$

$$E_B(k) = a_< \varepsilon^{1/2} V_A^{1/2} k^{-11/3} k_{\eta_{al}}^{13/6} \exp\left[-\frac{4}{3} + \frac{3}{2}\left(\frac{k_{\eta_{al}}}{k_{\mu_{un}}}\right)^{4/3} \right] \exp\left[-\frac{3}{2}\left(\frac{k}{k_{\mu_{un}}}\right)^{4/3} \right] \quad \text{for } k \geq k_{\eta_{al}}$$

Inertially aligned turbulence with Pm < 1



Fit of visco-resistive dissipation range spectrum to exponentially decaying MST spectrum yields inferred resistivity

MST spectrum has 1 decade of wavenumber variation ⇒ good fit with 2 parameter model

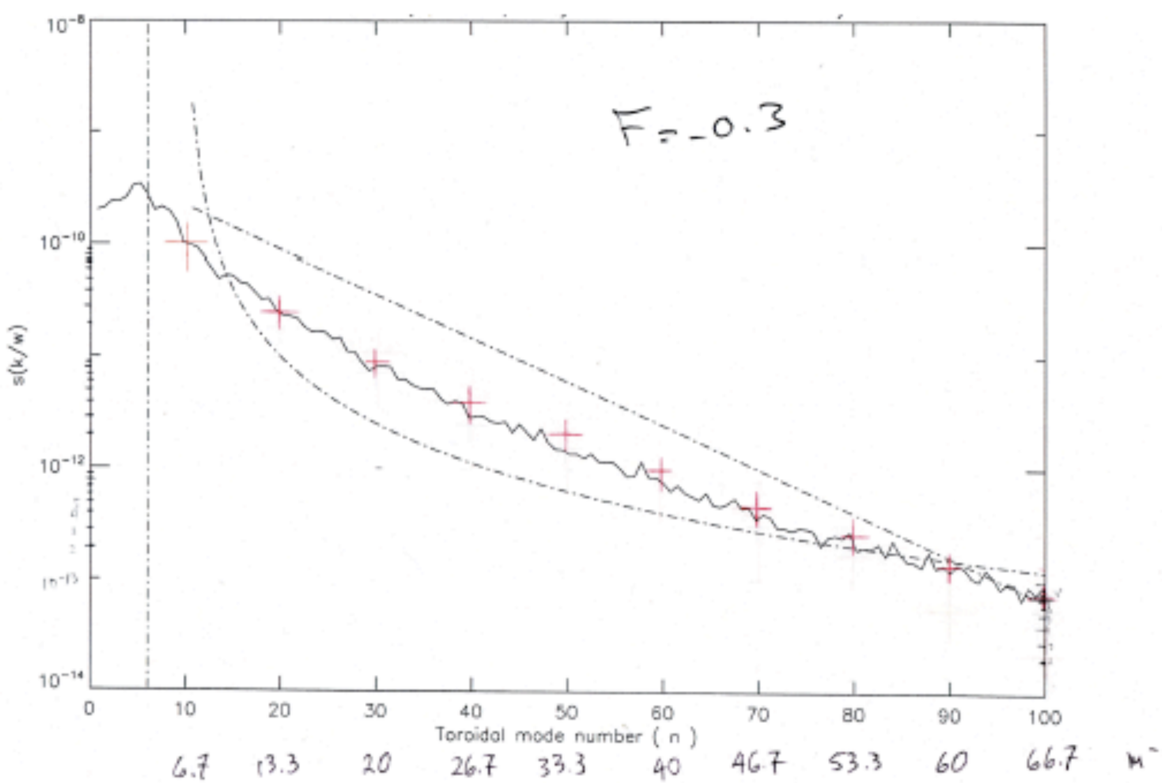
From fit: $\eta = 83\ m^2/s$

True value smaller by factor of 100

⇒Large noncollisional dissipation

Energy loss comparable to anomalous ion heating

ICR ion heating is good candidate for dissipation mechanism



Comments and discussion

Dissipation range spectra for resistive/viscous MHD have been derived

Alfvénicity, alignment, anisotropy, nonlocality have been examined

Tennekes Lumley closure too crude to treat Pm > 1

Spectra for Pm = 1, Pm < 1, with and without alignment in inertial scales combine basic forms $k^{-5/3}\exp(-k^{4/3})$ (unaligned) and $k^{-3/2}\exp(-k^{3/2})$ (aligned)

There are up to four dissipation wavenumbers

Comparison of dissipation range spectra with MST spectra indicate that there is a dissipation process that is much stronger than collisional resistivity/viscosity

Examination of magnetic shear suggests process is cyclotron damping