Decaying Kinetic Alfvén Wave Turbulence and Pulsar Scintillations

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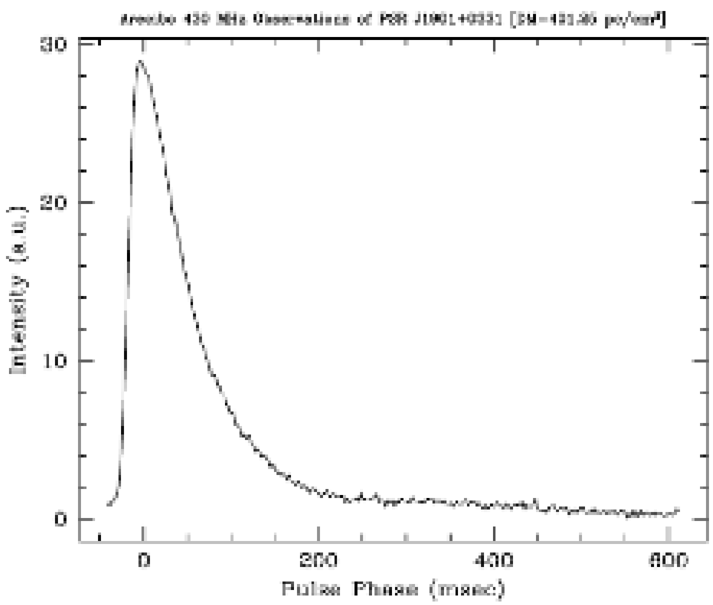
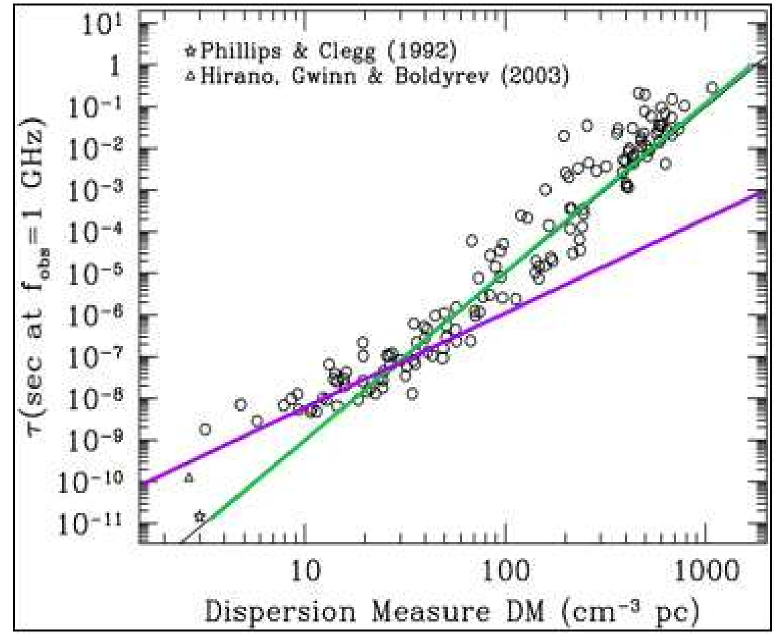
Motivation





- Pulsar temporal signal broadening constrains the distribution of electron density fluctuations in the ISM.
- Conventional models assumed a Gaussian-distributed n_e fluctuations, with relatively uniform 'step sizes' between signal refraction events.
- Gaussian assumption yields a signal time width $\tau \sim DM^2$, with DM the dispersion measure.
- Observation yields $\tau \sim DM^4$ for distant pulsars.
- Indicates non-Gaussian n_e fluctuations in ISM.

Motivation

- Boldyrev & Konigl (2006) postulated density fluctuations that are Lévy distributed $\Rightarrow$ enhanced tails and non-integrable distribution.
- Lévy distributed n_e could recover observed scalings; the signal broadening is dominated by one large refraction event, rather than many small events.
- Also argued that the dominant density fluctuations responsible are small-scale, $10^8 - 10^{10}$ cm.
- They proposed 2 models that could generate the proposed fluctuations, one of which emphasized MHD turbulence at scales near the ion gyroradius / ion sound-gyroradius where density fluctuations can become compressive.
- $\Rightarrow$ The Kinetic Alfvén Wave regime.

Kinetic Alfvén Model

$$\partial_t \psi + \nabla_{\parallel} \phi = \eta J + \nabla_{\parallel} n \tag{1}$$

$$\partial_t \nabla_{\perp}^2 \phi - \nabla \phi \times z \cdot \nabla \nabla_{\perp}^2 \phi = -\nabla_{\parallel} J \tag{2}$$

$$\partial_t n - \nabla \phi \times z \cdot \nabla n + \nabla_{\parallel} J = \mu \nabla_{\perp}^2 n \tag{3}$$

$$\nabla_{\parallel} = \partial_z + \nabla \psi \times z \cdot \nabla \qquad J = \nabla_{\perp}^2 \psi$$

- Full three-field system includes (1) Ohm's law with parallel electron compressibility, (2) vorticity evolution, and (3) electron continuity. [1]
- If electron density is eliminated, (1) and (2) become reduced MHD. If the magnetic field fluctuations become negligible and the electron density is Boltzmann distributed, (2) and (3) combine to form the Hasegawa-Mima equation for drift-wave turbulence.
- The kinetic Alfvén wave model is the regime where the electrostatic potential becomes negligible; Ohm's law and electron density combine. This regime is dominant at small scales.

[1] Terry, P.W. et. al., 2001, Phys. Plasmas, Vol. 8, No. 6

Reduced Kinetic Alfvén Wave Model

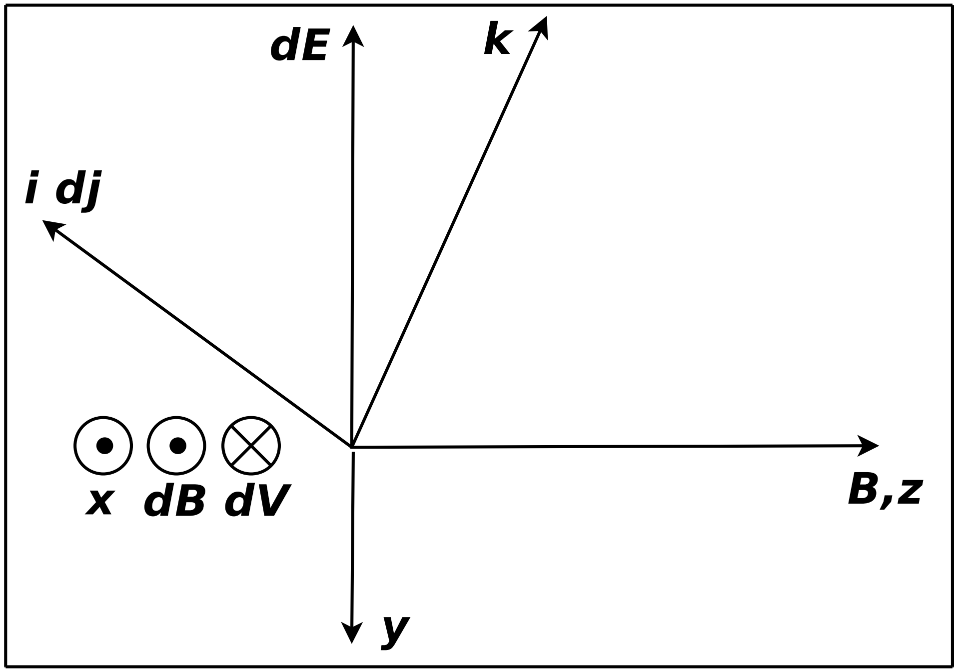
$$\partial_t \psi = \alpha \nabla_{\|} n + \eta J \qquad \text{Ohm's Law}$$

$$\partial_t n = -\nabla_{\|} J + \mu \nabla^2 n \qquad \text{Electron Continuity}$$

$$\nabla_{\|} = \partial_z + \nabla\psi \times \hat{\mathbf{z}} \cdot \nabla \qquad J = \nabla^2 \psi \, \hat{\mathbf{z}} \qquad \mathbf{B} = \mathbf{B}_0 + \nabla\psi \times \hat{\mathbf{z}}$$

- Model of small-scale n_e and B fluctuations with $L \leq \rho_s$. For the ISM, $L \sim 10^{10}\mathrm{cm}$.
- Ions form a neutralizing background.
- Ion flow is not dynamically active in this small-scale regime; ϕ equation drops from the usual 3-field equations, reducing to the system above.
- Electric field is inductive and is balanced by parallel electron density gradients.
- Diffusion in density is ad-hoc, allows control of η/μ ratio.

KAW Physics



(Hollweg 1999)

- Shear Alfvén Wave with large $k_\perp$ component.
- $\delta j_y \| \delta E_y \Rightarrow$ ion polarization drift.
- $\delta V_y \| k_y \Rightarrow \delta n_i \Rightarrow \delta p_i$

- $\delta n_i / n_0 = (k_y v_A / \omega_{ci}) \delta V_x / v_A$
- Magnetization drift in x, $\Rightarrow \delta B_z \Rightarrow$ coupling to slow wave.
- Electrons respond along magnetic field to $\delta n_i \Rightarrow \delta E_z$.
- Two field model captures electron compressibility and $\delta B_\perp$.

Large amplitude structures in KAW turbulence

- We simulate decaying KAW turbulence; our focus is on the kinds of nonlinear scattering structures that emerge and under what conditions they arise.
- The structures are dependent on the kind of damping.
- Our system here considers just fluid damping, and different ratios of the damping parameters are used.
- How can large-amplitude structures form and persist in the midst of turbulent fields?
- What kinds of statistics do large-amplitude structures generate?
- Two damping regimes, each in turn:
  - $\eta \sim \mu$ – both very small, large $\perp$ gradients.
  - $\eta \gg \mu$ – larger η smooths fields but sheets persist.

Simulation

- Two fields (n_e and ψ) are evolved in domains of size $2\pi \times 2\pi$.
- Resolution of 512^2.
- System integrated in Fourier space, with fully dealiased pseudospectral explicit fourth-order Runge-Kutta time stepping for the nonlinearities.
- Linear damping is handled via an integrating factor $\Rightarrow$ integrated exactly, with no timestepping constraints.
- All timestep constraints stem from nonlinear terms.
- Initial Conditions:
  - n_e and $k\psi$ are initialized with either random or correlated phases.
  - Internal energy E_I and magnetic energy E_M are equipartitioned, where

$$E_I = \sum_k n_k^2 \qquad \text{and} \qquad E_M = \sum_k k^2 \psi_k^2.$$

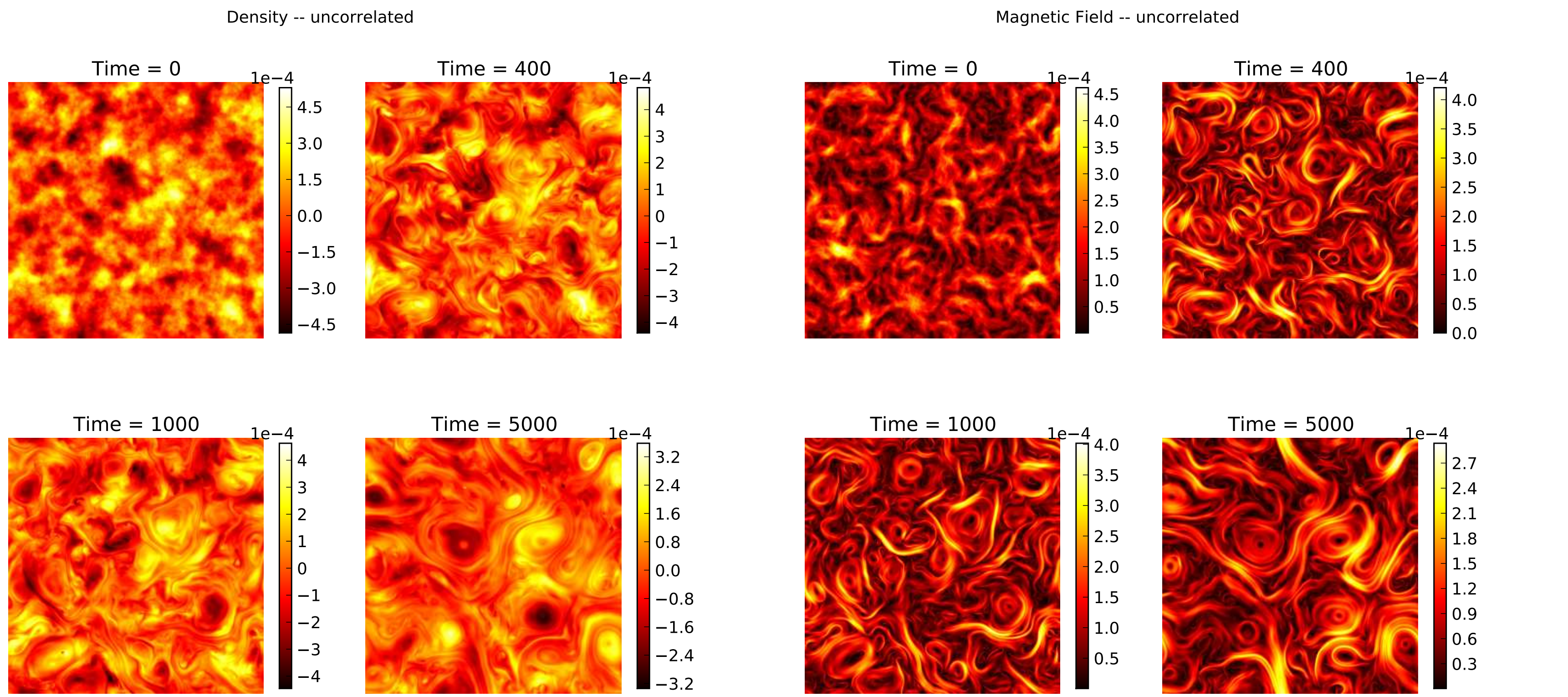
  - Spectra $\sim k^{-3}$ with peak in spectrum $k_0 \sim 6$.
  - Two damping regimes presented here: $\eta \sim \mu$, minimally damped to preserve simulation stability, and $\eta \gg \mu$.

Structure formation

(Movies of initial phase)

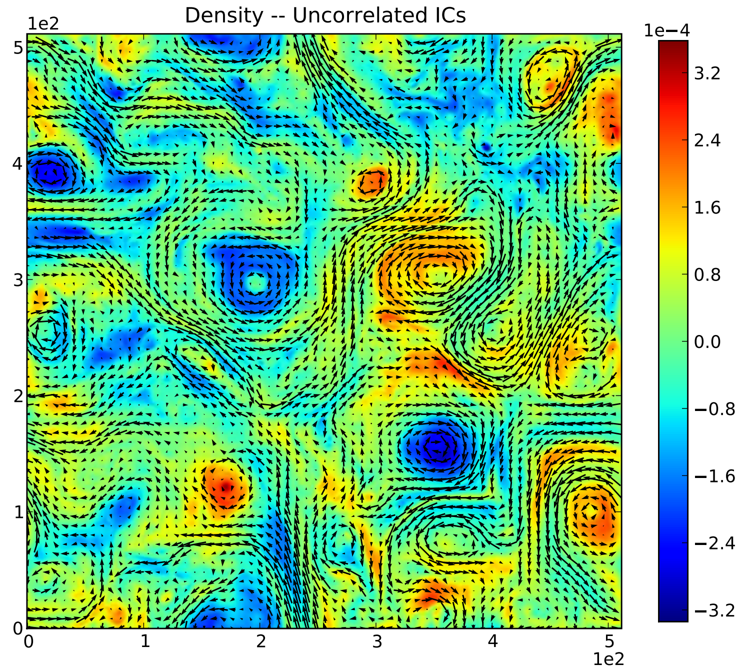
- Initial fields are statistically homogeneous and Gaussian.
- Some regions have overall enhancement and generate magnetic shear.
- Smaller fluctuations are sheared by larger ones.
- Large structures stabilize, turbulence localized to interface between large structures.
- Structure mergers begin after this phase.

Long-term evolution



- after initial structure formation phase, the structures merge. Turbulence remains localized to the interface between structures.
- At the center of many structures is a large-amplitude current filament, provided the damping parameters allow its persistence.
- Large-amplitude sheets can form at the interface between structures.

n_e - B Spatial Correlation



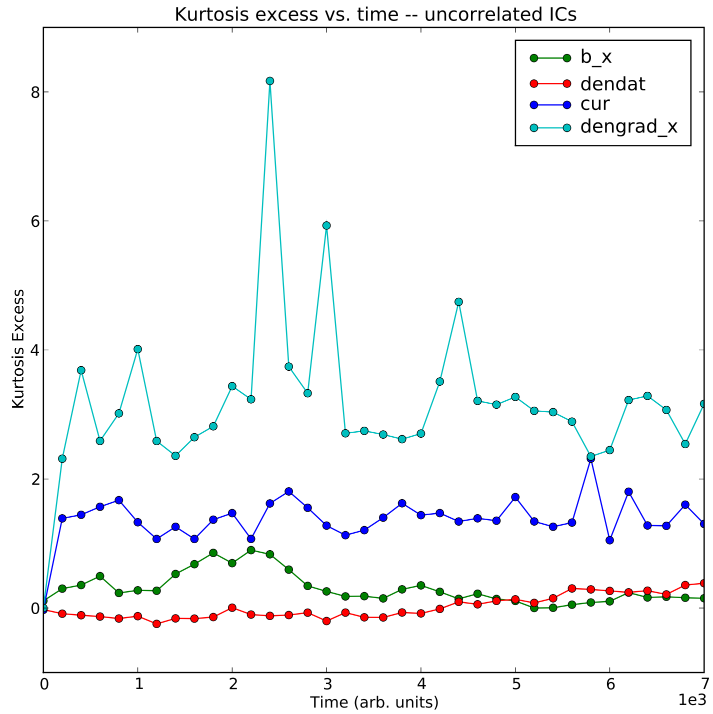
- n_e - B structure alignment apparent for phase-uncorrelated initial conditions.
- Suggests that system is insensitive to initial conditions, since spatial structure correlations spontaneously form from uncorrelated initial conditions.

$\eta \sim \mu$ – Sheets & Filaments

(Movies of ∇n here w/sheets)

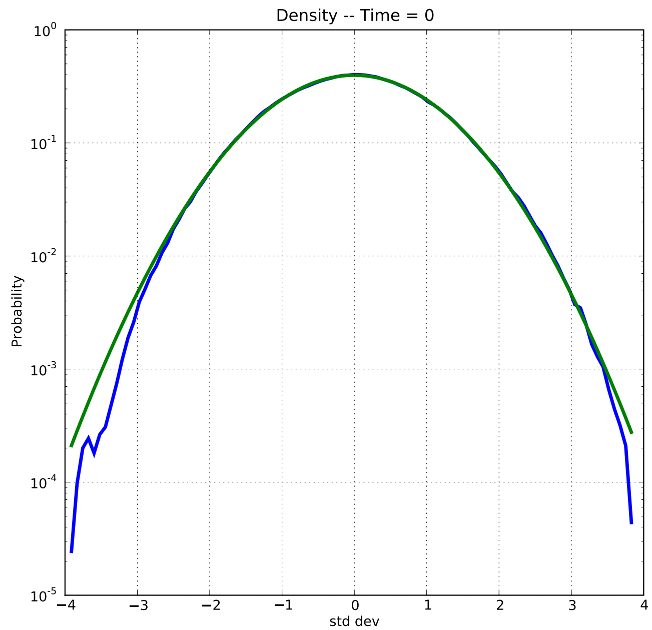
- varied structure in perp plane.
- large amp filaments in current.
- large amp sheets & filaments in den; large amp sheets in grad-den.
- Structures in $\perp$ plane give rise to strongly non-Gaussian fields.

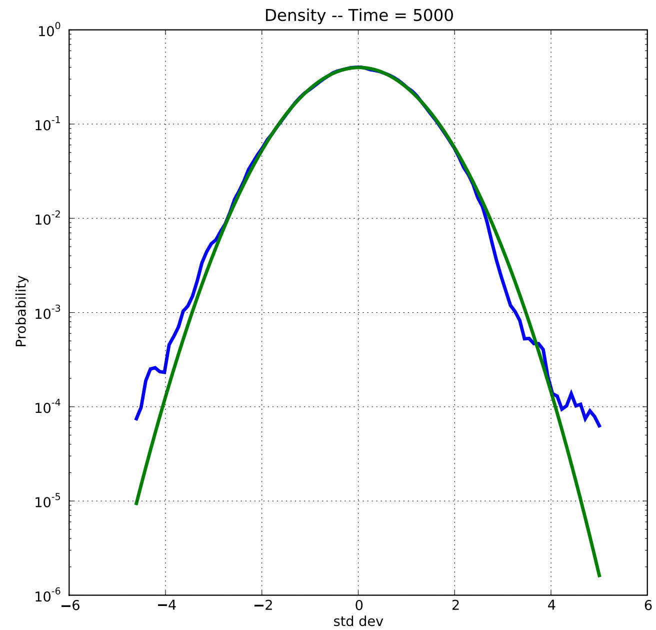
Non-Gaussian Statistics, $\eta \sim \mu$

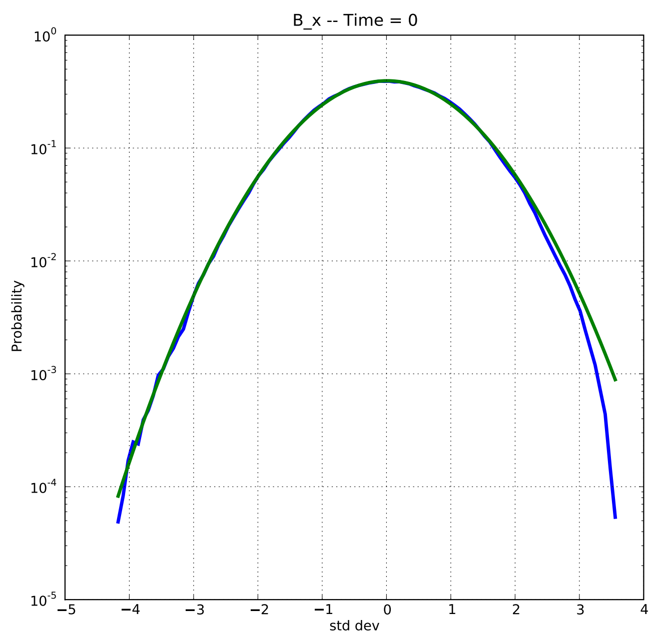


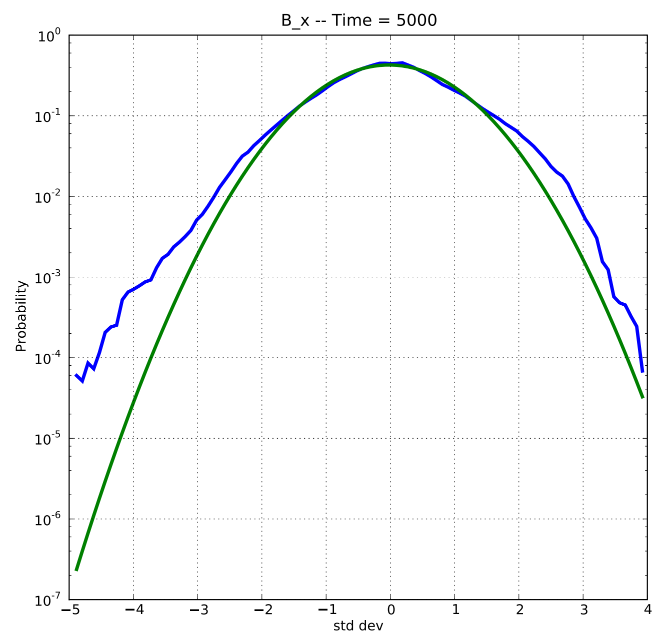
- Kurtosis excess vs. time for phase-uncorrelated initial conditions reveal non-Gaussian statistics for current and ∇n_e fields.
- both B and n_e fields more nearly Gaussian.
- As long as B and n_e fields have small deviation from Gaussian statistics, current and ∇n_e would be expected to be non-Gaussian. The fact that $K(\nabla n_e) > K(J)$ unanticipated.

Simulation Statistics: PDFs, $\eta \sim \mu$

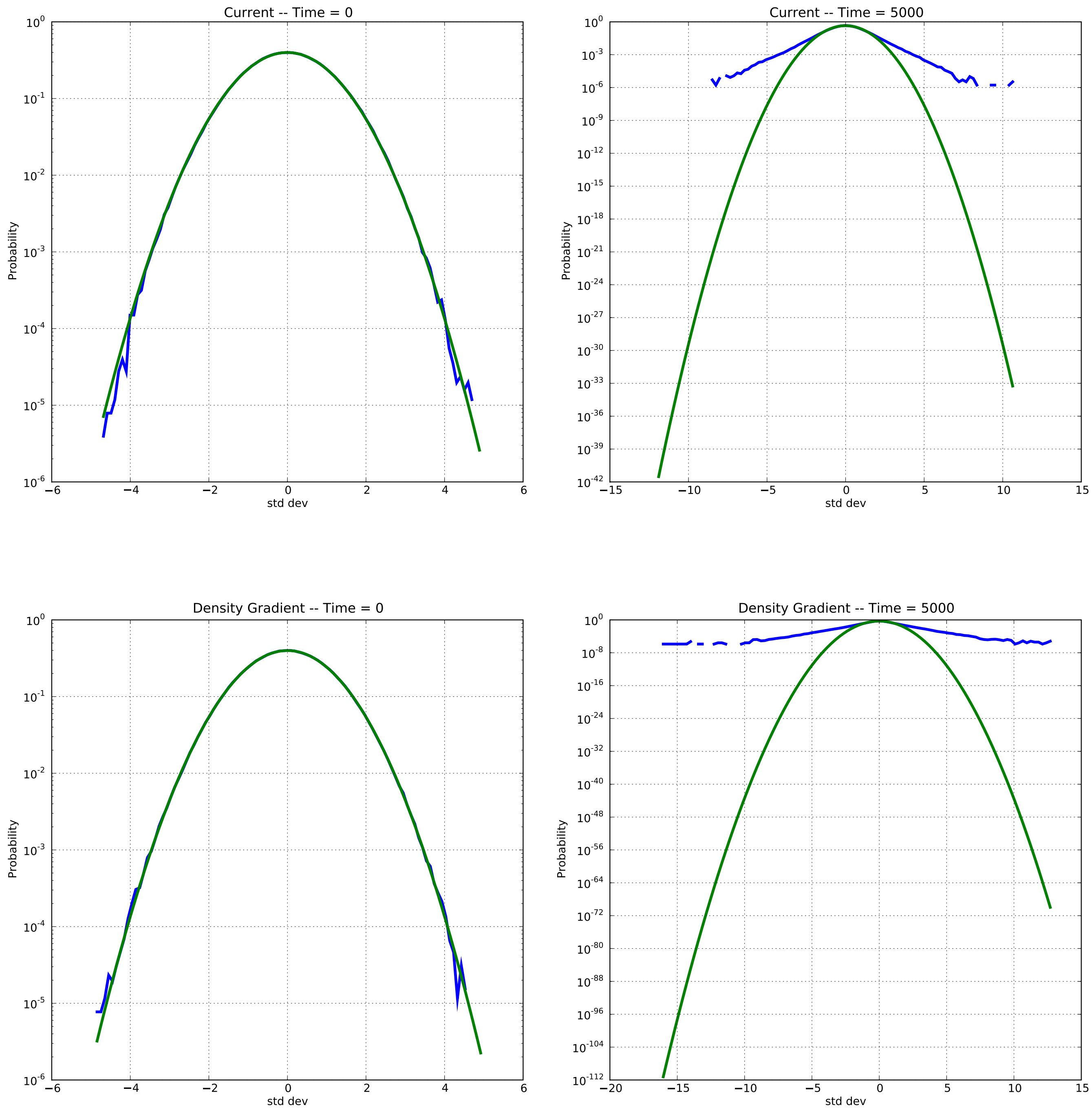




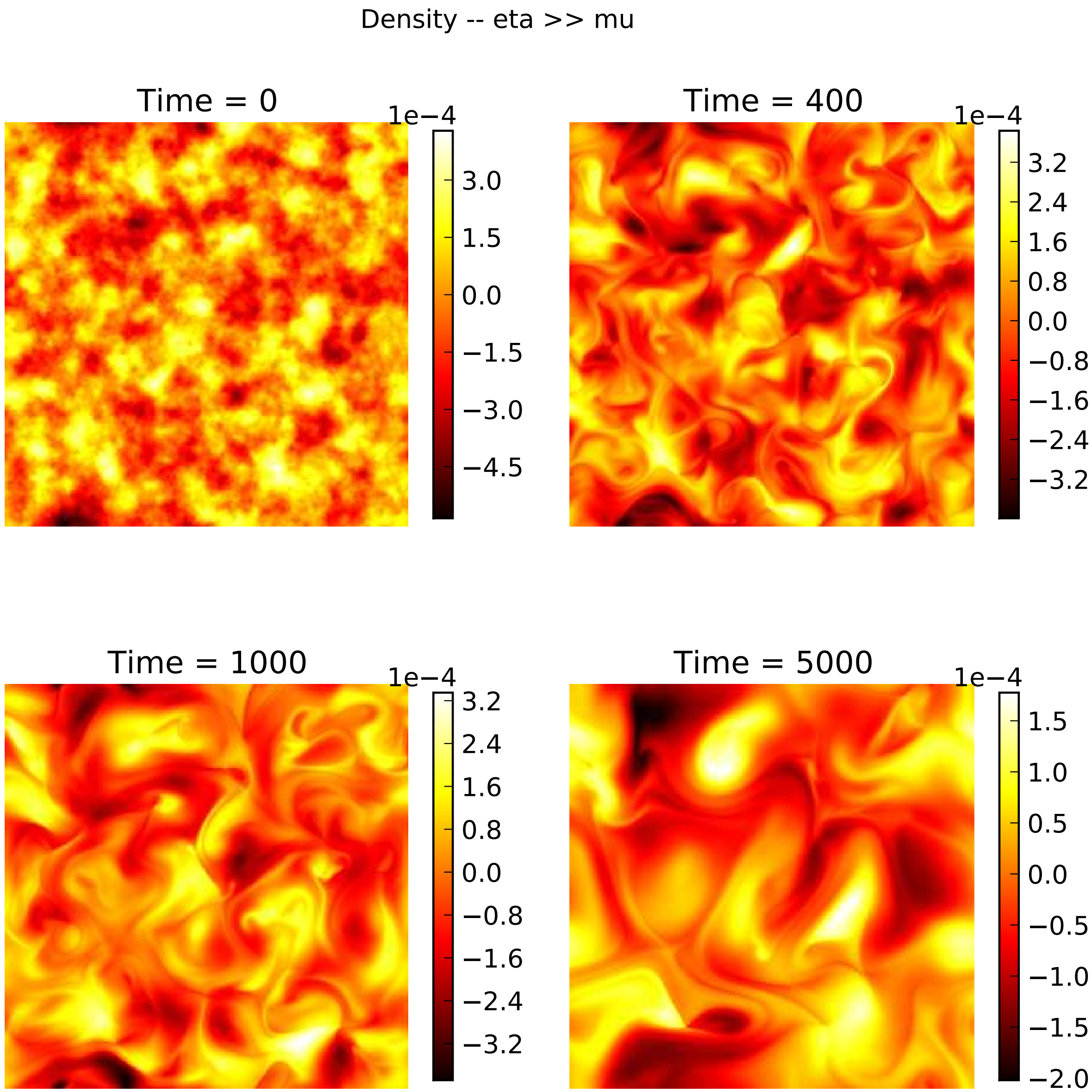




Simulation Statistics: PDFs, $\eta \sim \mu$

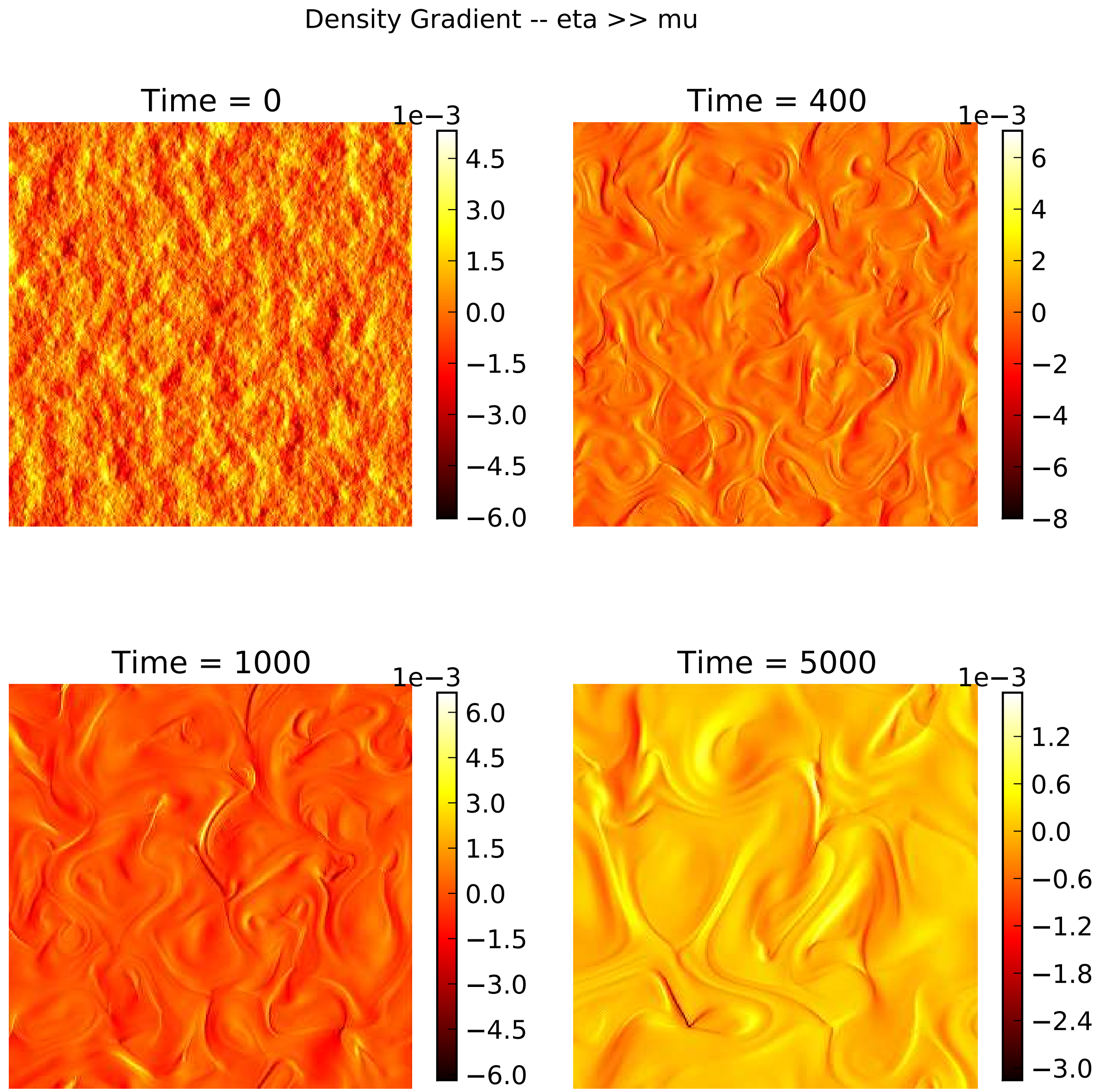


$\eta \gg \mu$ regime – Sheets only



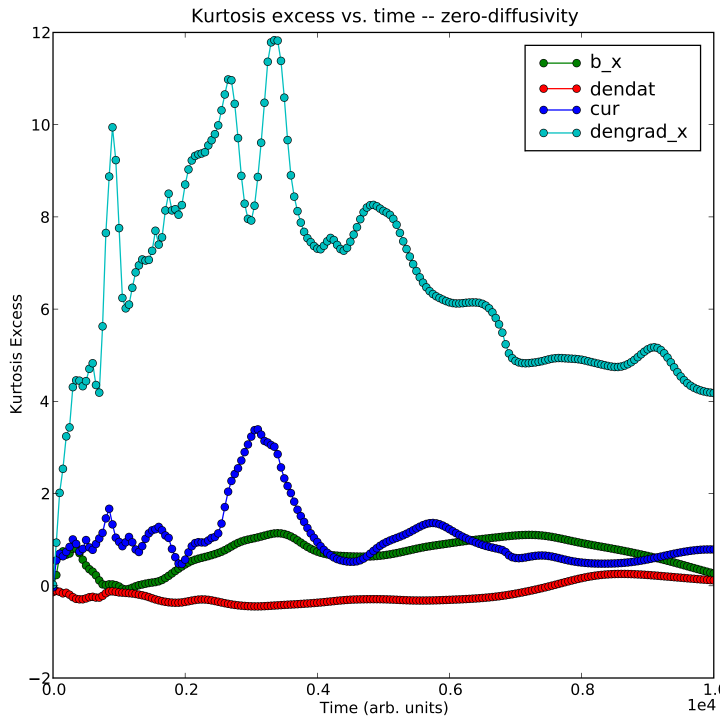
- Different diffusion regime damps circular density structures, but preserves sheet-like structures.
- Filaments unable to persist, broadened. Interface between flux tubes can still have large gradients $\Rightarrow$ density gradient sheets.

$\eta \gg \mu$ regime – density gradients still large



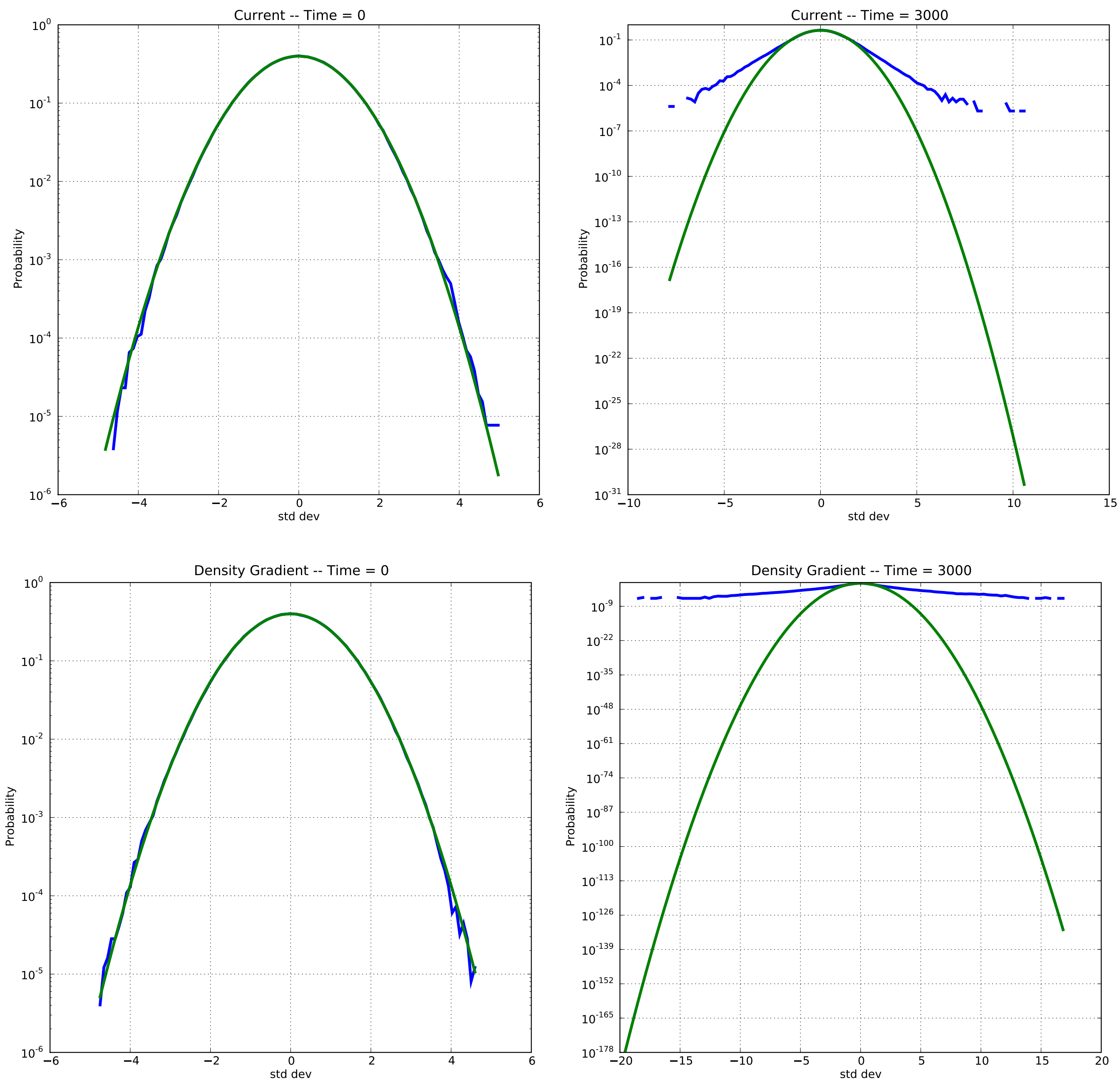
- Gradients still large; emphasize the sheet-like structures in density.

$\eta \gg \mu$ Regime

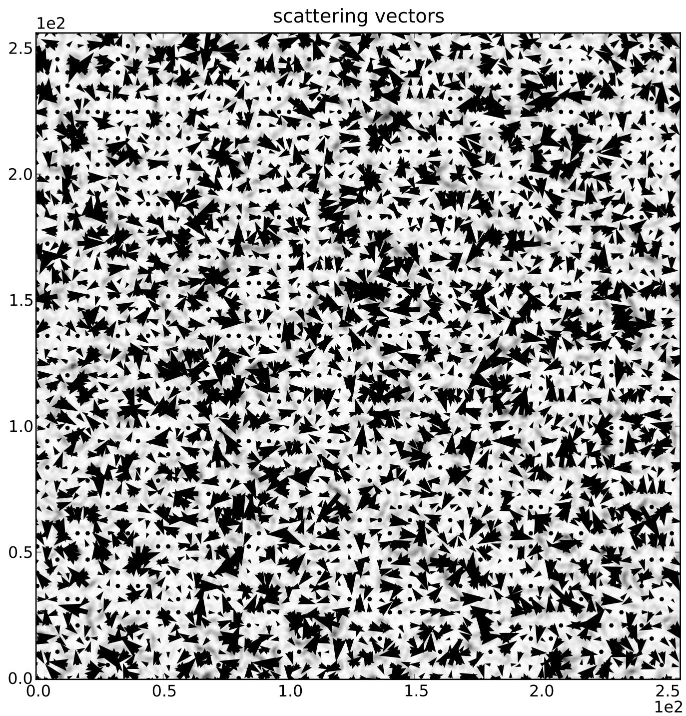


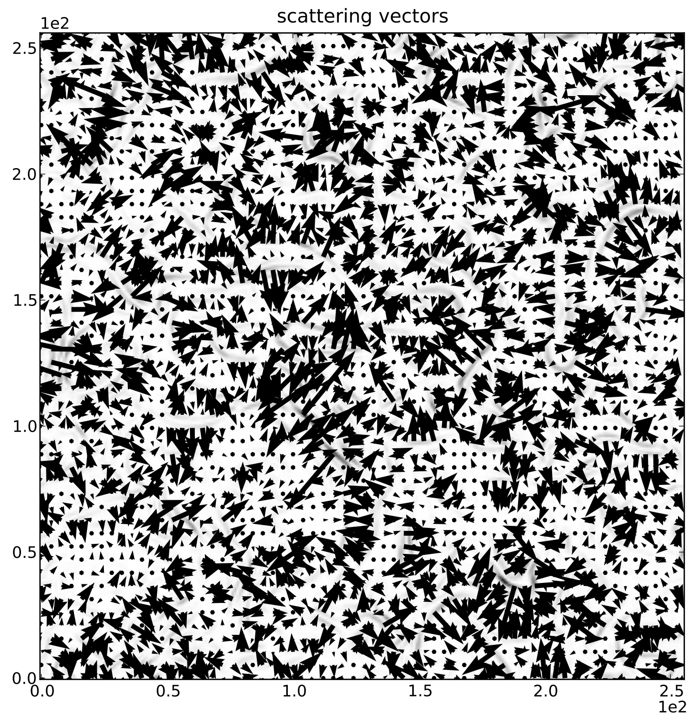
- Density gradient kurtosis reveals deviation from Gaussian statistics in this damping regime. Other fields consistent with $\eta \sim \mu$ damping regime.
- As long as μ small, non-Gaussian density gradients are robust to variation in η.

$\eta \gg \mu$ Regime: PDFs



Ray Tracing Studies – Preliminary



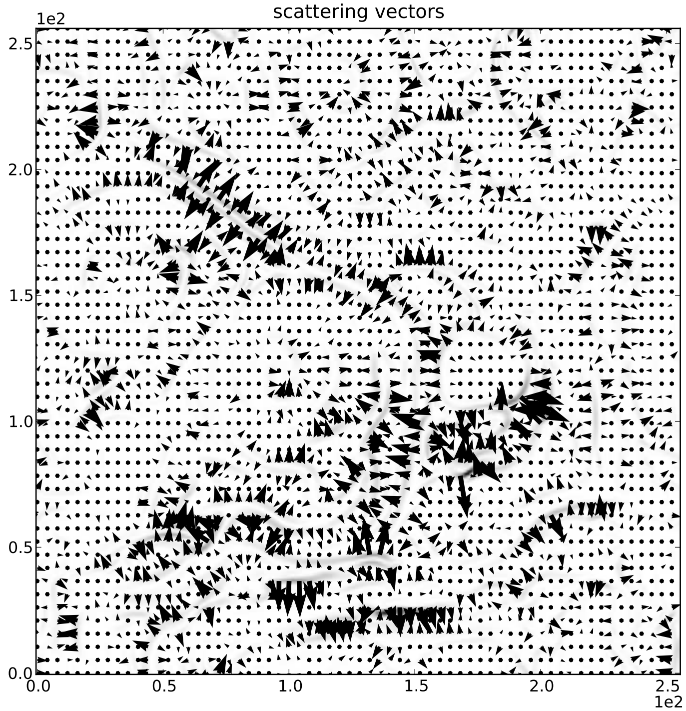


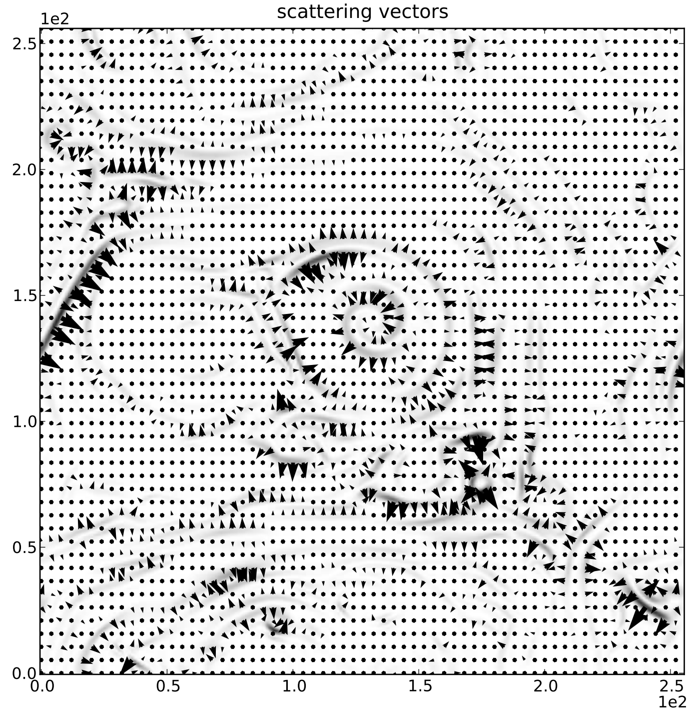
Overlaid vectors are $d\mathbf{n}/dt$ for density field's initial conditions.

Background field is $|\nabla n|$ – spatial correlation between vector field & $|\nabla n|$ will become clear.

Large scattering events spread uniformly throughout domain.

Ray Tracing Studies

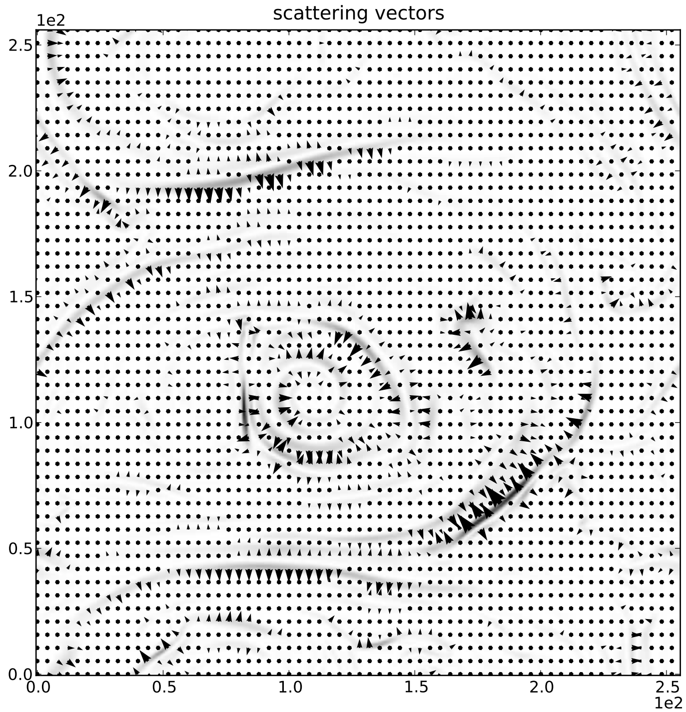


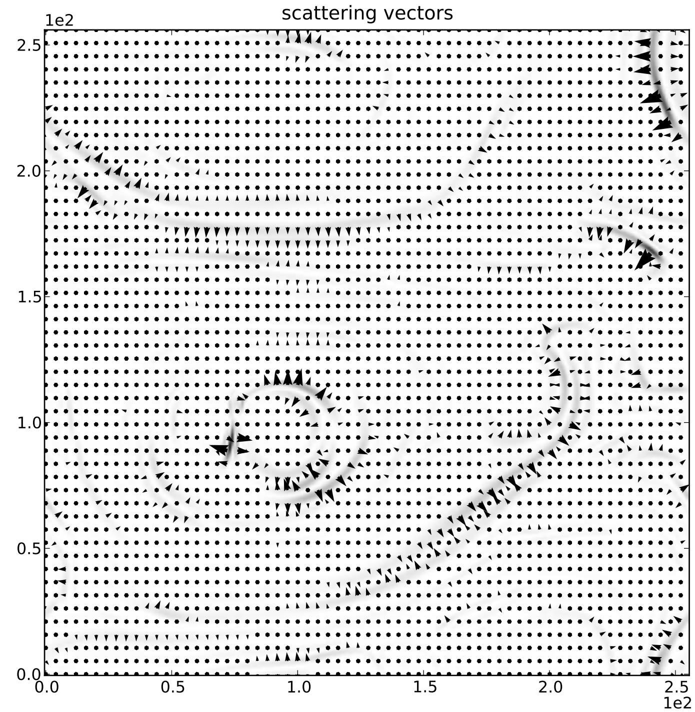


Same as previous; intermediate times.

Scattering events are spread-out, spatial correlation between vector field and $|\nabla n|$ evident.

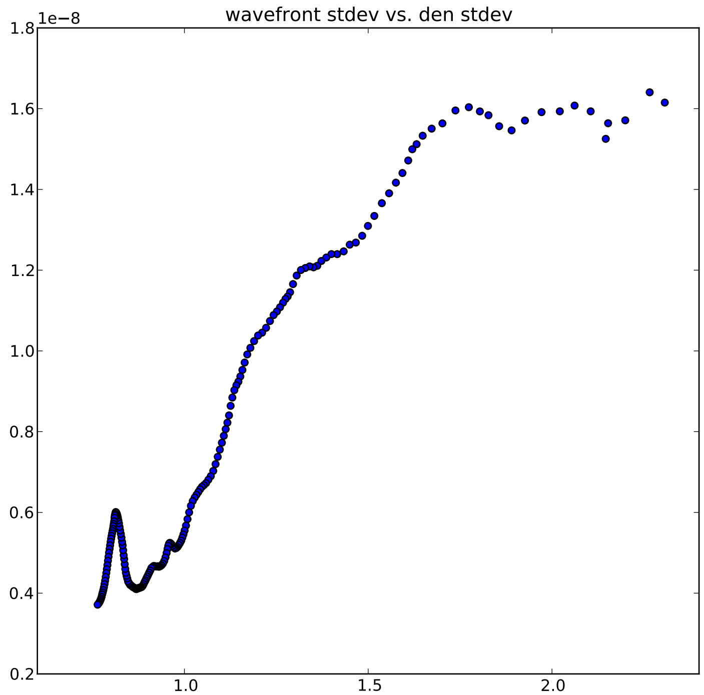
Ray Tracing Studies

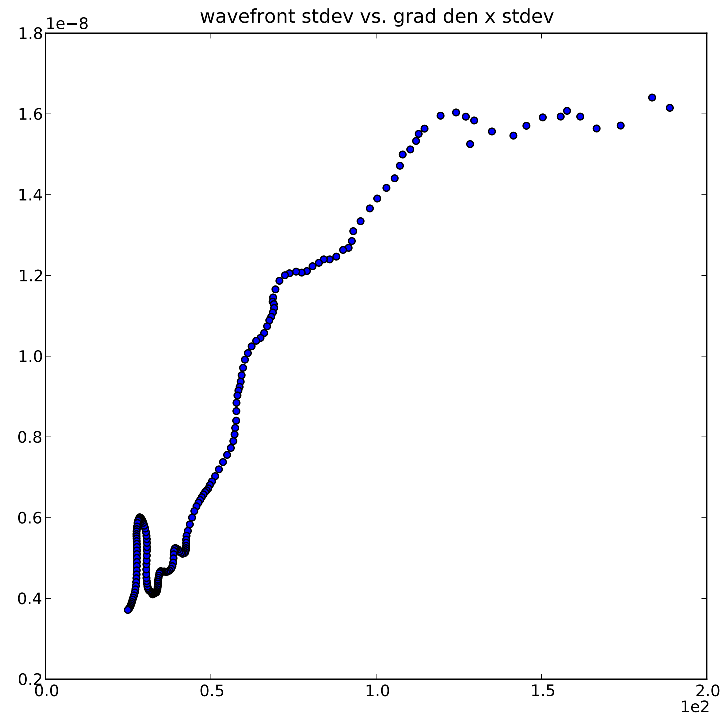




These timeslices have fairly large values of density gradient kurtosis (11 or so).

Ray Tracing Studies



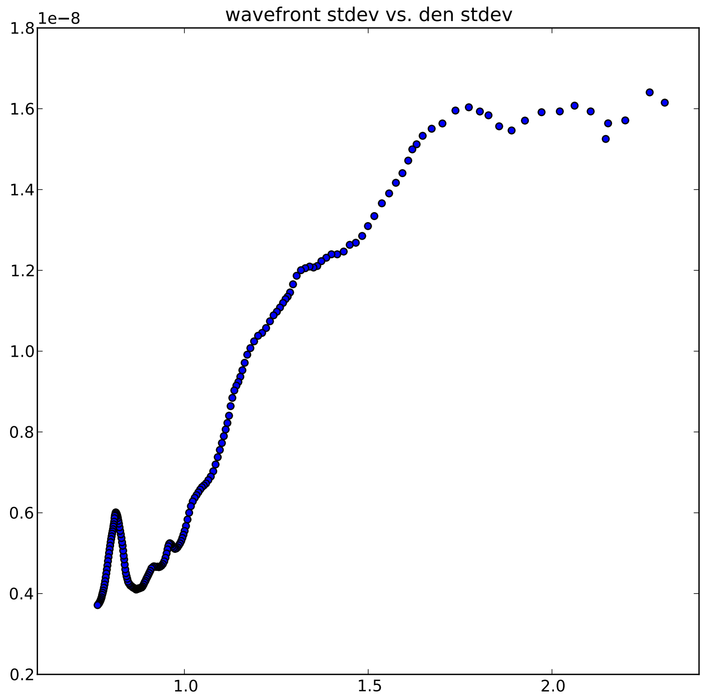


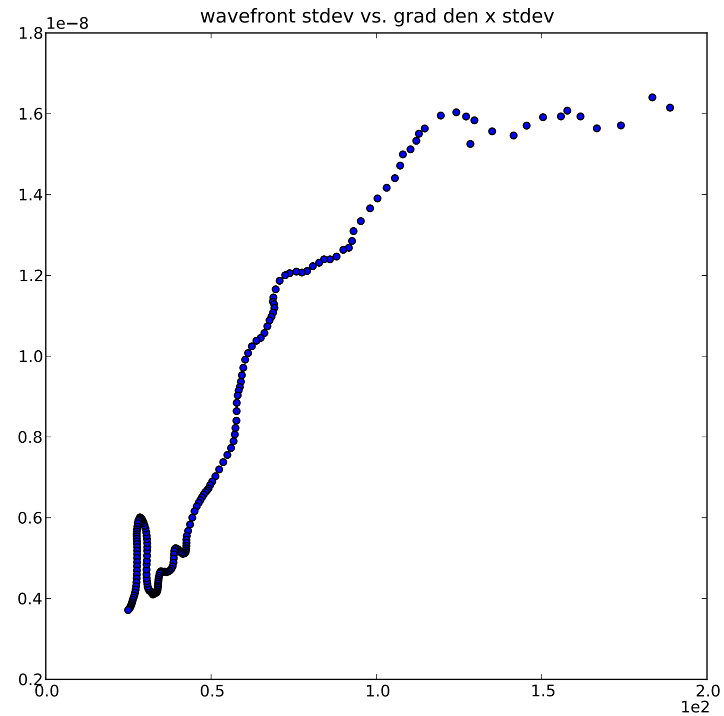
Standard deviation of wavefront spatial/temporal spread vs. stdev in density field (left) and $\partial n/\partial x$ (right).

Time advances from upper-right to lower-left, as overall amplitude of density field decays.

Strong correlation between spread in wavefront and fluctuation amplitude.

Ray Tracing Studies





Very crudely – two stages are discernible:

1. Initial stage, gaussian statistics, density structures arranging themselves and separating from each other. Wavefront broadening is independent of field amplitude.
2. Later stages, non-Gaussian statistics, density structures separated. Broadening more sensitive to field amplitude.

Interesting behavior in lower-left of plot, need more work to pin down.

Questions?