

## DETECTION OF [O I] $\lambda$ 6300 EMISSION FROM THE DIFFUSE INTERSTELLAR MEDIUM

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### ABSTRACT

The Wisconsin H $\alpha$  Mapper facility was used to obtain spectra of [O I]  $\lambda$ 6300 in three directions that sample the warm ionized component of the interstellar medium at the Galactic midplane and at  $z \approx -300$  pc. Weak interstellar [O I] emission was clearly detected toward all three directions, with [O I]  $\lambda$ 6300/H $\alpha$  intensity ratios for individual radial velocity components that range from less than 0.01 to approximately 0.04. According to photoionization models of the warm ionized medium, these [O I]/H $\alpha$  ratios suggest that most of the H $\alpha$  originates from density-bounded, nearly fully ionized regions along the lines of sight rather than from partially ionized H I clouds or layers of H II on the surfaces of H I clouds.

*Subject headings:* ISM: atoms — ISM: clouds — ISM: general — ISM: H II regions

### 1. INTRODUCTION

Although warm ionized hydrogen is a significant component of the interstellar medium (e.g., Reynolds 1993; Kulkarni & Heiles 1987), the nature of this gas and the source of its ionization are not yet clear. It has been proposed, for example, that the diffuse H $^+$  is the fully ionized portion of a pervasive warm intercloud medium (e.g., Miller & Cox 1993), that the H $^+$  is confined to H II–H I interfaces on the surfaces of clouds (McKee & Ostriker 1977), and that much of the H $^+$  is mixed with the neutral hydrogen, forming partially ionized, primarily neutral regions (Spitzer & Fitzpatrick 1993; Sciama 1990).

It is now possible to begin to test ideas about the relationship between the H $^+$  and H $^0$  through observations of the extremely faint [O I]  $\lambda$ 6300 emission line. The intensity of the interstellar [O I] line with respect to H $\alpha$  is a sensitive probe of the ionization state of the emitting gas (e.g., Weisheit 1977; Domgörgen & Mathis 1994) because the large H $^+$  + O $^0$   $\leftrightarrow$  H $^0$  + O $^+$  charge exchange cross section couples the ionization fraction of the atomic oxygen to that of the hydrogen (Field & Steigman 1971). Since the [O I] emission is the result of collisional excitations by thermal electrons, its intensity is a measure of the neutral hydrogen content within the warm ionized (i.e., H $\alpha$ -emitting) regions. In particular, the volume photon emissivity  $\epsilon$  of the O  $^1D$   $\lambda$ 6300 transition relative to H $\alpha$  is related to the hydrogen ionization ratio  $n(\text{H}^+)/n(\text{H}^0)$  through the relation

$$\frac{\epsilon_{\text{OI}}}{\epsilon_{\text{H}\alpha}} = 2.63 \times 10^4 \frac{n(\text{H}^0)}{n(\text{H}^+)} \xi \frac{n(\text{O})}{n(\text{H})} \times \frac{T_4^{1.85}}{1 + 0.605T_4^{1.105}} \exp\left(-\frac{2.284}{T_4}\right), \quad (1)$$

where  $n(\text{O})/n(\text{H})$  is the gas-phase abundance of oxygen,  $T_4$  is the electron temperature in units of  $10^4$  K, and  $\xi$  is a factor with a value near unity given by  $(1+r)/(\frac{8}{9}+r)$ , where  $r = n(\text{H}^0)/n(\text{H}^+)$ . Equation (1) incorporates the temperature dependence of the [O I] collision strength given by Berrington & Burke (1981) and Péquignot (1990).

If the hydrogen in the warm ionized medium (WIM) were 50% ionized, then the [O I]/H $\alpha$  emissivity ratio would be 0.5 for a gas-phase oxygen abundance of  $3 \times 10^{-4}$  (Cardelli & Meyer 1997) and a temperature of  $10^4$  K. An upper limit of

0.02 was determined from an earlier study of one sight line at  $l = 114^\circ$ ,  $b = 0^\circ$  (Reynolds 1989), which indicated a high fractional ionization for the WIM hydrogen and placed interesting constraints on the relationship between interstellar H II and H I in photoionization models (see, for example, Domgörgen & Mathis 1994). Observations with the new, much more sensitive Wisconsin H $\alpha$  Mapper (WHAM) spectrometer are presented below; these observations repeat the original sight line mentioned above and provide information about the [O I]/H $\alpha$  intensity ratio in two additional directions, one of which samples the interstellar medium away from the Galactic midplane.

### 2. INSTRUMENTATION

The observations were carried out with the Wisconsin H $\alpha$  Mapper (WHAM), which is a recently completed facility funded by the National Science Foundation for the detection and study of faint optical emission lines from the diffuse interstellar medium (Tufté 1997; Reynolds et al. 1997). WHAM consists of a 15 cm aperture, dual-etalon Fabry-Perot spectrometer coupled to a dedicated 0.6 m telescope, and this provides a  $1^\circ$  diameter beam on the sky and produces a  $12 \text{ km s}^{-1}$  resolution spectrum across a  $200 \text{ km s}^{-1}$  wide spectral window. The spectral window can be set to any wavelength between 4800 and 7200 Å, and a CCD camera serves as a multichannel detector, recording the spectrum as a classical Fabry-Perot interference pattern, or “ring spectrum.” Compared with the instrument used for much of our previous work on WIM emission lines, WHAM can obtain the same signal-to-noise ratio in 1/100th the integration time.

WHAM is currently located at Kitt Peak National Observatory and operated remotely from Madison, WI. Through mid-1998, it will be dedicated to one of its primary tasks, a sky survey of the distribution and kinematics of the diffuse interstellar H II. The observations described below were obtained when WHAM was located at Pine Bluff Observatory near Madison as part of its initial testing phase.

### 3. OBSERVATIONS

Observations of interstellar [O I] are difficult because of the [O I]  $\lambda$ 6300 airglow line, which is of order 100 times brighter than the interstellar line. Fortunately, in certain directions at particular times of the year, it is possible to resolve these two

sources spectrally. For example, the diffuse ionized gas in the Perseus spiral arm in the Galactic longitude range  $100^\circ < l < 140^\circ$  is observed to be Doppler-shifted by approximately  $-40 \text{ km s}^{-1}$  ( $0.8 \text{ \AA}$ ) because of the differential rotation of the Galactic disk (e.g., Reynolds 1983; also see Joncas, Roger, & Dewdney 1989 and references therein), and from July through September the Earth's orbital velocity adds an additional shift of about  $25 \text{ km s}^{-1}$  in this region of the sky. This  $-65 \text{ km s}^{-1}$  shift with respect to the Earth allows WHAM to resolve cleanly the interstellar [O I] in the Perseus arm from the much brighter terrestrial line.

Spectra of [O I]  $\lambda 6300$  and  $H\alpha$  were obtained toward three diffuse background directions:  $l = 114^\circ, b = 0^\circ$  on the night of 1996 July 18–19, and  $l = 130^\circ, b = 0^\circ$  and  $l = 130^\circ, b = -7.5^\circ$  on 1996 September 11–12. The first two directions were selected because they sample regions of relatively bright background emission (Reynolds 1983) and because previous observations revealed high [S II]/ $H\alpha$  line intensity ratios characteristic of emission from the WIM (Reynolds 1985). Also,  $(114^\circ, 0^\circ)$  is the direction in which an upper limit was set on the [O I] intensity with the previous, less sensitive spectrometer (Reynolds 1989). The third direction was selected to sample gas above the Galactic midplane ( $|z| \approx 330\text{--}400 \text{ pc}$  at the  $2.5\text{--}3 \text{ kpc}$  distance of the Perseus arm). For comparison, spectra were also obtained toward a bright O star H II region (NGC 7000) on 1996 July 22–23.

The diffuse background observations were carried out in the manner described by Reynolds (1989), in which the “on-source” observations named above were alternated with [O I] and  $H\alpha$  observations toward higher Galactic latitude “off-source” directions where the intensity of the interstellar  $H\alpha$  emission, and presumably the interstellar [O I], is much weaker. For  $114^\circ, 0^\circ$  the off-source directions were  $120^\circ, +12.5^\circ$  and  $108^\circ, -12.5^\circ$ . For  $130^\circ, 0^\circ$  and  $130^\circ, -7.5^\circ$  the off-source directions were  $130^\circ, \pm 15^\circ$ . Each [O I] observation had a total integration time of 1800 s on-source and 1800 s off-source. Each  $H\alpha$  observation had a total integration time of 60 s “on” and 60 s “off.” The observations toward NGC 7000 had integration times of 60 s for the [O I] and  $H\alpha$  with no off spectrum for the  $H\alpha$ .

The resulting spectra for  $114^\circ, 0^\circ$  are shown in Figure 1, which plots the signal versus the radial velocity with respect to the local standard of rest (LSR). The top two panels in Figure 1 show the on spectrum (*diamonds*) and the off spectrum (*solid line*) for the  $H\alpha$  and the [O I]. In the  $H\alpha$  spectra, the geocoronal line is the relatively narrow emission component at  $+23 \text{ km s}^{-1}$ , while the interstellar  $H\alpha$  is the broader, more complex profile composed of emission from both local gas near the LSR and more distant gas in the Perseus arm at  $-40 \text{ km s}^{-1}$  (see also Reynolds 1983). This interstellar  $H\alpha$  is quite prominent in the on (i.e.,  $114^\circ, 0^\circ$ ) spectrum. In the off spectrum, it is weak, appearing only as an extended blue wing on the geocoronal line. At  $114^\circ, 0^\circ$  the  $H\alpha$  emission near the LSR is severely blended on its blue side with the more intense interstellar emission associated with the Perseus arm and on its red side with the geocoronal line. The [O I] spectra are dominated by two terrestrial lines, [O I] at  $+25 \text{ km s}^{-1}$  and OH at  $-85 \text{ km s}^{-1}$ . In Figure 1 (*bottom two panels*), the  $H\alpha$  and [O I] difference spectra obtained by subtracting the higher Galactic latitude off-source spectrum from the corresponding on-source spectrum at  $114^\circ, 0^\circ$  are also included. This subtraction greatly reduces the magnitude of the terrestrial features common to both directions relative to the interstellar emission at  $114^\circ, 0^\circ$ . In the  $H\alpha$  difference spectra, the most prominent feature is the  $-40 \text{ km s}^{-1}$  component associated with the diffuse ionized gas

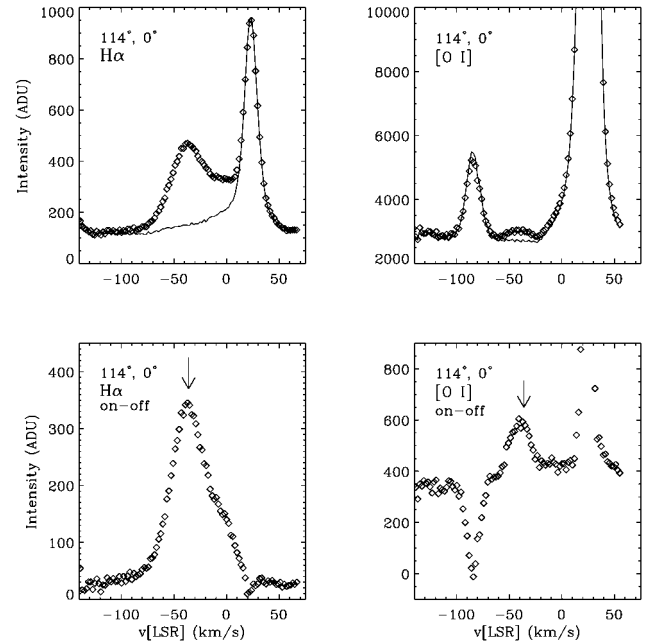


FIG. 1.— $H\alpha$  and [O I]  $\lambda 6300$  spectra for  $l = 114^\circ, b = 0^\circ$ . The top two panels show the average spectrum for the on-source direction (*diamonds*) and the off-source directions (*solid line*). The bottom two panels show the difference between the on-source and off-source spectra on expanded vertical scales. The arrows denote the radial velocity of the  $H\alpha$  emission associated with the Perseus arm at  $114^\circ, 0^\circ$ . The sampling is at  $2 \text{ km s}^{-1}$  intervals; the FWHM of the spectrometer's velocity resolution is  $12 \text{ km s}^{-1}$ .

in the Perseus arm. A corresponding interstellar emission component is also clearly present in the [O I] difference spectrum between the imperfectly subtracted terrestrial components. Note the greatly expanded ordinate scale of the [O I] difference spectrum. The terrestrial line near  $0 \text{ km s}^{-1}$  prevents the clear identification of any [O I] emission component near the LSR.

Figure 2 shows the  $H\alpha$  and [O I] difference spectra for  $130^\circ, 0^\circ$  (*upper panels*) and  $130^\circ, -7.5^\circ$  (*lower panels*). Interstellar [O I] is again clearly detected in both directions at the radial velocity of the interstellar  $H\alpha$  emission associated with the Perseus arm. In the  $130^\circ, 0^\circ$  difference spectrum, no  $H\alpha$  appears near  $0 \text{ km s}^{-1}$  because the intensity of the emission near the LSR in the off spectrum is approximately the same or even a little brighter than in the on spectrum.

The Perseus arm  $H\alpha$  emission profile toward  $130^\circ, -7.5^\circ$  is much broader than the  $H\alpha$  profile associated with the Perseus arm emission toward  $114^\circ, 0^\circ$  and  $130^\circ, 0^\circ$ . It appears to be a blend of two radial velocity components, one at about  $-31 \text{ km s}^{-1}$  and a second at  $-60 \text{ km s}^{-1}$  (indicated by the two arrows). On the other hand, the [O I] difference spectrum shows no evidence for the bluer portion (the  $-60 \text{ km s}^{-1}$  component) of the emission that is clearly present in the  $H\alpha$  spectrum. These  $130^\circ\text{--}7.5^\circ$  spectra therefore reveal a significant variation in the [O I]/ $H\alpha$  intensity ratio for different velocity components along the same sight line in the diffuse background. The  $H\alpha$  emission component near the LSR also could have an associated feature in the [O I] spectrum, but again the terrestrial [O I] line prevents a clear identification of a low-velocity interstellar [O I] component.

#### 4. RESULTS

To determine [O I]  $\lambda 6300/H\alpha$  intensity ratios, Gaussian components were fitted to the emission profiles in the difference spectra. Fits to the [O I] spectra are the principal source of

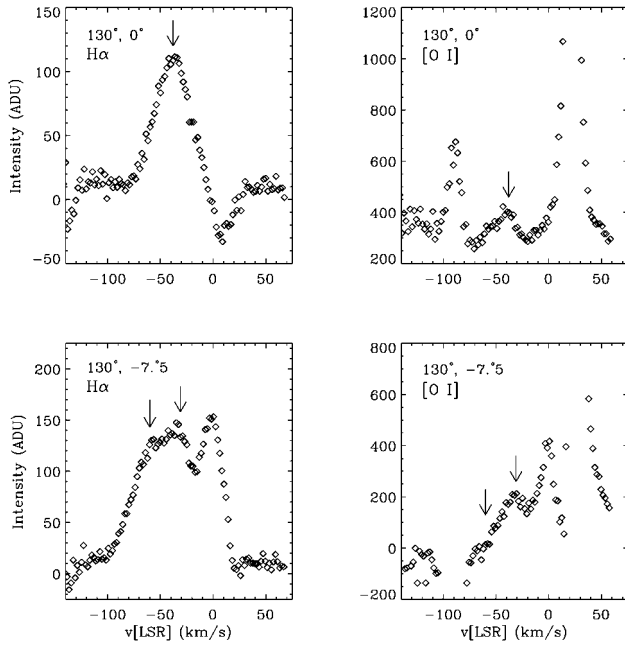


FIG. 2.— $H\alpha$  and [O I]  $\lambda 6300$  difference spectra showing the interstellar emission at  $l = 130^\circ$ ,  $b = 0^\circ$  (upper panels) and  $l = 130^\circ$ ,  $b = -7.5^\circ$  (lower panels). The arrows denote the radial velocity of the  $H\alpha$  emission components associated with the Perseus arm. The gap at  $-90 \text{ km s}^{-1}$  in the [O I] difference spectrum for  $130^\circ$ ,  $-7.5^\circ$  (lower right panel) is due to an oversubtraction of the terrestrial OH line.

uncertainty in the derived ratios because these spectra have relatively low signal-to-noise ratios and because their baselines are less well defined as a result of incomplete subtractions of the two relatively strong terrestrial emission lines. This uncertainty was estimated by fitting each spectrum with different baselines and by observing the goodness of fit for various values of the Gaussian's position, width, and area. A correction was also made for the small differences in the instrument's optical transmittance and detector quantum efficiencies between 6300 and 6563 Å.

The results of these fits are presented in Table 1, which lists the values and estimated uncertainties for the radial velocity, width, and intensity for the Perseus arm  $H\alpha$  emission components as well as the estimates for the velocity and width of the corresponding [O I] emission. The resulting [O I]/ $H\alpha$  line intensity ratio is also listed, followed by the hydrogen ionization ratio calculated from equation (1) for electron temperatures of 6000 and 10,000 K. Velocity components near the LSR are not included because of the large baseline uncertainty there, which is due to the imperfectly subtracted terrestrial [O I] emission near  $0 \text{ km s}^{-1}$ , except for NGC 7000, where the nebular [O I] intensity is an order of magnitude larger than

that in the diffuse background. The number in parentheses in the width column for [O I] is the expected width of the [O I] line based on the observed  $H\alpha$  line width, the O/H mass ratio, and the assumptions that the gas temperature is 8000 K and the [O I]-emitting and  $H\alpha$ -emitting atoms are well mixed. For  $130^\circ$ ,  $-7.5^\circ$  free fits of the widths of the  $H\alpha$  components and the widths and positions of the [O I] components were not possible because of the complexity of these spectra. The [O I] results in this direction are based on the assumption that the interstellar [O I] components have radial velocities equal to those of the  $H\alpha$  components and widths as calculated above. While these assumptions are questionable (see § 5), they at least make it possible to estimate the [O I]/ $H\alpha$  intensity ratio for the  $-31 \text{ km s}^{-1}$  component and to set an upper limit on this ratio for the  $-60 \text{ km s}^{-1}$  component.

## 5. DISCUSSION AND CONCLUSIONS

Diffuse interstellar [O I]  $\lambda 6300$  emission is clearly detected in each of the three sight lines that sample the warm ionized medium in the Perseus spiral arm. The  $-40 \text{ km s}^{-1}$  component toward  $114^\circ$ ,  $0^\circ$  has an [O I]/ $H\alpha \approx 0.02$ , just at the upper limit set with the earlier, less sensitive instrument (Reynolds 1989). In the two other directions, the observed ratios for the Perseus arm velocity components appear to range from less than 0.01 to about 0.04. Diffuse [O I] emission also could be associated with more local interstellar gas near the LSR, but blending with the much stronger terrestrial [O I] line prevents a clear identification of these lower velocity components. Improved techniques in the collection and subtraction of on-source and off-source spectra (e.g., a more rapid sampling of the on and off directions as well as a larger number of off sight lines associated with every on sight line) may significantly reduce the residual terrestrial contamination in future interstellar [O I] studies.

These low [O I]/ $H\alpha$  intensity ratios suggest a high fractional ionization of hydrogen within the diffuse  $H\alpha$ -emitting regions, although not as high as in the bright O star H II region, NGC 7000, where the intensity ratio is nearly a factor of 10 lower. According to equation (1), a [O I]/ $H\alpha$  ratio of 0.04 corresponds to hydrogen ionization ratios of 1.2 and 13, for temperatures of 6000 and 10,000 K, respectively, and [O I]/ $H\alpha < 0.01$  has corresponding ionization ratios  $n(\text{H}^+)/n(\text{H}^0) > 5$  and  $> 50$ , respectively. For NGC 7000, the ionization ratios are 17 and 170 for 6000 and 10,000 K. The derived values for  $n(\text{H}^+)/n(\text{H}^0)$  are clearly sensitive to the temperature of the gas, which unfortunately is not well determined. A temperature near 8000 K is indicated by the widths of  $H\alpha$  and [S II]  $\lambda 6716$  emission line profiles in the diffuse background (Reynolds 1985, 1988), which is consistent with a lower limit of 5500 K derived from forbidden line intensities (see Reynolds 1993). A more accurate determination will probably have to await the detection of the extremely weak [N II]  $\lambda 5755$  line, whose intensity relative to

TABLE 1  
RESULTS

DIRECTION	$H\alpha$			[O I]		$I([\text{O I}])/I(H\alpha)$ (energy units)	$n(\text{H}^+)/n(\text{H}^0)$	
	$v$ ( $\text{km s}^{-1}$ )	$I$ ( $R$ )	$w$ (FWHM)	$v$ ( $\text{km s}^{-1}$ )	$w$ (FWHM)		6000 K	10,000 K
$114^\circ, 0^\circ$ .....	$-36 \pm 1$	$9.8 \pm 2.0$	$34 \pm 1$	$-40 \pm 1$	$19 \pm 5$ (27)	$0.020 \pm 0.003$	2.7	28
$130^\circ, 0^\circ$ .....	$-38 \pm 1$	$3.3 \pm 0.7$	$35 \pm 2$	$-42 \pm 1$	$18 \pm 5$ (28)	$0.028 \pm 0.009$	2.0	20
$130^\circ, -7.5^\circ$ .....	$-60 \pm 3$	$3.4 \pm 0.7$	$36 \pm 4$	(-60)	(29)	$< 0.012$	$> 4.7$	$> 48$
	$-31 \pm 2$	$2.5 \pm 0.5$	25 <sup>a</sup>	(-31)	(14)	$0.044 \pm 0.011$	1.2	13
NGC 7000 .....	$-0.4 \pm 1.0$	800	$23.7 \pm 0.3$	$4.5 \pm 1.0$	$15.2 \pm 0.3$ (11.5)	$0.0033 \pm 0.0003^b$	17	170

<sup>a</sup> Fixed at this value during the fitting.

<sup>b</sup> Error dominated by [O I] and  $H\alpha$  calibration uncertainties.

the [N II]  $\lambda$ 6584 line is highly temperature sensitive (Osterbrock 1989).

These observations place significant constraints on photoionization models and the relationship between the diffuse H II and the H I. For example, the derived hydrogen ionization ratios for the WIM are much higher than that deduced for the warm ionized hydrogen toward the Galactic halo star HD 93521 by Spitzer & Fitzpatrick (1993). For the ionization fraction,  $n(\text{H}^+)/n(\text{H}^0) = \frac{1}{3}$ , and temperature, 6000 K, deduced by Spitzer & Fitzpatrick, the predicted value for [O I]  $\lambda$ 6300/H $\alpha$  = 0.16, 4–13 times larger than the observed ratios and upper limit listed in Table 1. Therefore, if these three Perseus arm sight lines are fair samples of the interstellar medium, such partially ionized regions must represent only a small fraction of the WIM. Moreover, according to models by Domgörgen & Mathis (1994) and Bland-Hawthorn, Freeman, & Quinn (1997), ratios as low as those observed toward 114°, 0° and the  $-60 \text{ km s}^{-1}$  component toward 130°,  $-7^\circ 5'$  require that a large fraction of the H $\alpha$  originate from density-bounded regions, semitransparent to the ionizing radiation. This would mean that most (80% in Domgörgen & Mathis's "composite" model) of the H $\alpha$  in these cases cannot be from ionized cloud edges as pictured, for example, in the McKee & Ostriker (1977) model.

The observed variations in the intensity ratio from sight line to sight line and even between different velocity components along the same sight line (130°,  $-7^\circ 5'$ ) imply inhomogeneous ionization and/or temperature conditions within the WIM. For example, a variable mix of density-bounded regions (low

[O I]/H $\alpha$ ) and ionization-bounded H II–H I interfaces (higher [O I]/H $\alpha$ ) could produce such variations (Domgörgen & Mathis 1994). If different [O I]/H $\alpha$  intensity ratios are present among separate regions that are blended into a single, unresolved velocity component, then, even though all of the components are contributing to the H $\alpha$ , only one or two may be dominating the [O I] emission. This could explain why the best-fit radial velocities of the [O I] are not always coincident with the H $\alpha$  (within the estimated uncertainties) and why the measured [O I] line widths differ from the predictions (given in parentheses in Table 1) that are based on the H $\alpha$  width. A comparison of H $\alpha$  and 21 cm maps has shown that at least some H $\alpha$  is associated both spatially and kinematically with H I clouds (Reynolds et al. 1995). Therefore, it will be interesting in future work to explore whether these H $\alpha$ -emitting H I clouds have significantly higher [O I]/H $\alpha$  ratios than those found in this study, that is, whether they have ratios consistent with the predictions for ionization-bounded H II–H I interfaces or perhaps even partially ionized H I clouds.

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#### REFERENCES

- Berrington, K. A., & Burke, P. G. 1981, *Planet. Space Sci.*, 29, 377  
 Bland-Hawthorn, J., Freeman, K. C., & Quinn, P. J. 1997, *ApJ*, 490, 143  
 Cardelli, J. A., & Meyer, D. M. 1997, *ApJ*, 477, L57  
 Domgörgen, H., & Mathis, J. S. 1994, *ApJ*, 428, 647  
 Field, G. B., & Steigman, G. 1971, *ApJ*, 166, 59  
 Joncas, G., Roger, R. S., & Dewdney, P. E. 1989, *A&A*, 219, 303  
 Kulkarni, S. R., & Heiles, C. 1987, in *Interstellar Processes*, ed. D. J. Hollenbach & H. A. Thronson, Jr. (Dordrecht: Reidel), 87  
 McKee, C. F., & Ostriker, J. P. 1977, *ApJ*, 218, 148  
 Miller, W. W., III, & Cox, D. P. 1993, *ApJ*, 417, 579  
 Osterbrock, D. E. 1989, *Astrophysics of Gaseous Nebula and Active Galactic Nuclei* (Mill Valley: University Science Books)  
 Péquignot, D. 1990, *A&A*, 231, 499  
 Reynolds, R. J. 1983, *ApJ*, 268, 698  
 Reynolds, R. J. 1985, *ApJ*, 294, 256  
 ———. 1988, *ApJ*, 333, 341  
 ———. 1989, *ApJ*, 345, 811  
 ———. 1993, in *AIP Conf. Proc.* 278, *Back to the Galaxy*, ed. S. S. Holt & F. Verter (New York: AIP), 156  
 Reynolds, R. J., Tufte, S. L., Haffner, L. M., Jaehnig, K., & Percival, J. W. 1997, *Publ. Astron. Soc. Australia*, in press  
 Reynolds, R. J., Tufte, S. L., Kung, D. T., McCullough, P. R., & Heiles, C. 1995, *ApJ*, 448, 715  
 Sciama, D. W. 1990, *ApJ*, 364, 549  
 Spitzer, L., Jr., & Fitzpatrick, E. 1993, *ApJ*, 409, 299  
 Tufte, S. L. 1997, Ph.D. thesis, Univ. Wisconsin, Madison  
 Weisheit, J. C. 1977, *ApJ*, 215, 755