Planetary Interiors/Size

- Apply the virial theorem \(2E_k = -E_p\)
- What's the kinetic energy?
  - Motion of electrons (degeneracy and electrostatic)
  - Protons don't contribute much at all
- What's the potential energy?
  - gravitational
Degeneracy Energy

- $M_p$ has $N_p$ atoms of average mass number, $A \rightarrow$ so $N_p = M_p/A_m$ $\rightarrow$ each atom has $ZN_p$ electrons
- Each electron occupies a volume with diameter, $d$, so that $d = (A_m/ZM_p)^{1/3}R_p$
- From quantum mechanics, $E_k = p^2/(2m_e)$ and $p\lambda = h$
- The de Broglie wavelength, $\lambda$, is the size of the electron volume so $\lambda = 2\pi d$ (longest possible wavelength)

Degeneracy Energy cont’d

- Put that altogether and get:
  - $E_k = (h^2/2m_e)(4\pi^2d^2)^{-1}$ per electron volume
  - Substitute expression for $d$, multiply by $ZN_p$ to get total degenerate energy

$$E_K = \gamma M_p^{5/3}Z^{5/3}A^{-5/3}R_p^{-2}$$
Electrostatic

- Assume non-relativistic
- \( E_e \sim \frac{1}{4\pi\varepsilon_0}(Ze^2/d) \) (per electron)
- Plug in \( d \) from previous page and multiply by \( N_pZ \) to get:

\[
E_e \sim \xi M_p^{4/3} Z^{7/3} A^{-4/3} R_p^{-1}
\]

Gravitational Energy

\[
E_g = -G(M_p^2/R_p)
\]
Combine all the energies....

- Use virial theorem so that \(2E_k = E_e + E_g\)
- Rearrange to get a relation between \(R_p\) and \(M_p\)

\[
R_p^{-1} = (\text{const})A^{1/3}Z^{2/3}M_p^{-1/3} + (\text{const})M_p^{1/3}A^{5/3}Z^{-5/3}
\]

- Peaks at \(\log(M) \sim 27\) (kg) and \(\log(R) \sim 8\) \(\rightarrow\) right around Jupiter!

Maximum radius

- Take \(dR_p/dM_p = 0\), solve for \(M_{R_{\text{max}}}(H)\)
- Get: \(M_{R_{\text{max}}} = (\text{const}) (Z^{7/3}/A^{4/3})^{3/2}\)
- Insert this in for the mass in the long equation and get:

\[
R_{\text{max}} = (\text{const}) Z^{1/2}/A
\]

- \(R_{\text{max}}(H) \sim 1.2 \times 10^8\) m
- The central pressure for a H body with maximum radius is about the pressure needed to ionize H.
Giant Planets – issues

- Basics
  - Interior structure (is there something solid?)
  - Sources of heat
  - Magnetic fields
  - Composition – deviations from solar?
  - Atmospheric physics
  - What accounts for the colors?
- Can the giant planet systems be considered "mini-solar systems?"
  - Ring systems
  - Extensive satellite systems
- Cool Moons
  - Io, Europa, Titan, Triton
- Formation
  - How long does it take to form a gas giant?
  - How did they get there?
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Giant Planets - Composition

- How do you measure the chemical composition of a giant planet?
How do you measure the chemical composition of a giant planet?
- Spectroscopy $\rightarrow$ largely IR, some optical, some radio
  - CO$_2$ absorbs at 4.3$\mu$m, 15$\mu$m
  - CH$_4$ at 3.3$\mu$m, 7.7$\mu$m, and numerous optical lines
  - NH$_3$
- Occultation
  - Planet comes between Earth and star; atmosphere absorbs light $\rightarrow$ radiative transfer!!!!
Giant Planets - Composition

- How ‘bout going there?
- Cassini (Saturn), New Horizons (Jupiter)
- Galileo Probe – Jupiter (December 1995)
  - Descended for ~57 mins
  - Final depth ~600 km, pressure ~24 bar
  - Measures composition, ρ, T, wind speed, P
- Results from Galileo (see table 4-5)
  - “All” H, He
  - He underabundant in atmosphere
  - CH$_4$ more abundant in U+N by factor of 10

Atmospheric Compositions

<table>
<thead>
<tr>
<th></th>
<th>Sun</th>
<th>Jup</th>
<th>Sat</th>
<th>Ura</th>
<th>Nep</th>
</tr>
</thead>
<tbody>
<tr>
<td>H$_2$</td>
<td>84</td>
<td>86.4</td>
<td>97</td>
<td>83</td>
<td>79</td>
</tr>
<tr>
<td>He</td>
<td>16</td>
<td>13.6</td>
<td>3</td>
<td>15</td>
<td>18</td>
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<td>H$_2$O</td>
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<td>0.1</td>
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<td>-</td>
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<tr>
<td>CH$_4$</td>
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<td>0.21</td>
<td>0.2</td>
<td>2</td>
<td>3</td>
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<tr>
<td>NH$_3$</td>
<td>0.02</td>
<td>0.07</td>
<td>0.03</td>
<td>-</td>
<td>-</td>
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<tr>
<td>H$_2$S</td>
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<td>0.008</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>C/H</td>
<td>1</td>
<td>2.9</td>
<td>3</td>
<td>30-40</td>
<td>30-40</td>
</tr>
</tbody>
</table>
What’s an atmosphere?

- What do you call the atmosphere of a gas ball?
- H$_2$O in atmospheres
  - Via mid-IR spectroscopy $\Rightarrow$ H$_2$O in all giant planets
  - 10x overabundant (over theory) $\Rightarrow$ accretion?
- Atmospheric/interior structure/composition
  - Stratosphere (P < 1 bar) H$_2$O, NH$_3$, CH$_4$
  - Troposphere (1 bar) $\Rightarrow$ clouds of NH$_3$, NH$_4$SH, H$_2$O
  - 10$^4$ > P > 100 bar $\Rightarrow$ cloud deck of SiO$_2$, CO, N$_2$
  - P $\sim$ 10$^4$ $\Rightarrow$ H$_2$
  - P $\sim$ 10$^6$ $\Rightarrow$ metallic H

Physical Processes in the Atmospheres of the Giant Planets

- Upper layers of atmosphere
  - Convection to P $\sim$ 100 mbar
  - T increases above this $\Rightarrow$ why?
  - Highest, thinnest $\Rightarrow$ charged particles trapped in B field
- Condensation
  - J+S $\Rightarrow$ Hydrated molecules (H$_2$O, H$_2$S, NH$_4$SH)
  - U+N $\Rightarrow$ CH$_4$ $\Rightarrow$ so why are these planets blue??
  - What accounts for the bands of color on J, S??
More Physical Processes

- Define some equation of state for H, He mixture....
- $T = T(P), P = P(\rho) \Rightarrow \rho(R)$
- Assume hydrostatic equilibrium $\rightarrow$ conclude the interior is nearly completely convective!!!
  - What are the conditions for convection?

Convective Interiors

- What does convection do?
Convective Interiors

- What does convection do?
  - Stirs things up → affects chemistry
  - Source of mechanical energy → drives eddies in outer layers of atmosphere
  - Lightning → Galileo satellite detected lightning in Jovian cloud structures

Special Note - Volatiles

- Volatiles must be trapped in a solid (e.g. ice)
- At high T, volatiles remain gaseous
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- At high T, volatiles remain gaseous

Galileo – Ar, Kr, Ne 2x more abundant than predicted by solar nebula model \( \rightarrow \) implies formation in cold region (T < 75K)
  - Ar, N\(_2\) at T < 30K
  - Ne at T < 17K \( \rightarrow \) way beyond Pluto!!!!
  - how’d this stuff get incorporated into Jupiter???

Structure of Giant Planets

- What happens to common molecules at ridiculous pressures?
- What kind of deformations occur with a spinning ball of gas?
- Are there solid cores in the centers of the giant planets?
Is Jupiter the biggest “planet” possible?
- $\sim 80M_J \rightarrow$ w/out H fusion = brown dwarf
- Probably have D fusion

What’s a planet?
Is Jupiter the biggest “planet” possible?
- ~80M_J → w/out H fusion = brown dwarf
- Probably have D fusion

What is the basic behavior of H?
- T(1 bar) → 165K (J), 135K (S), 76K (U), 50K (N)
- 10^5 bar → hot liquid
- 1Mbar → 10,000 km → liquid metallic H
  - Fantastic conductor → B-field
Comparison of the Giant Planets

Structure of the Giant Planets
A few specific studies

- Benedetti et al 1999 Science 286 100 → What is the fate of methane under conditions in Neptune’s interior?
- Anallotto et al 1997 Science 275 1288 → more on methane in Neptune, Uranus
- Cavazzoni et al 1999 Science 283 44 → How do NH$_3$ and H$_2$O behave at high P,T?